MANNHEIM SLIDE RULE.

CROCKETT.

AN EXPLANATION

OF THE

PRINCIPLES AND OPERATION

OF THE

MANNHEIM SLIDE RULE.

BY

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PREFACE.

This explanation of the principles of the slide rule is written primarily for the use of the students of the Institute and so presupposes an acquaintance with the principles of logarithms.

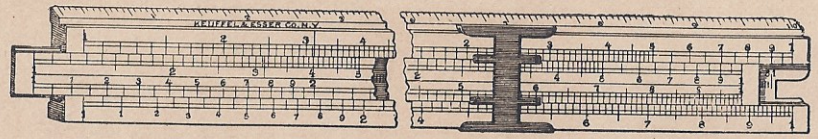
The attention of the reader is called particularly to Arts. 10, 11, 12, and 13. The three statements in Art. 13 should be memorized and used in all cases covered by them, for it is believed that in this way a beginner can quickly learn to use the instrument with facility. The possibility of the existence of the general method of Art. 13 was suggested by the rules given in the pamphlet accompanying the Thacher rule.

To follow the explanations it will be very convenient to have a rule or else to make one. Cut out two pieces of stiff paper or card-board about 12 inches long, one being 2 inches wide to form the rule and the other ½ inch wide to form the slide. Along the middle of the wider piece draw two lines ½ inch apart and lay off scale *A* (Art. 2) above one line and scale *D* below the other. On the upper margin of the narrower piece lay off scale *B* and on the lower, scale *C*. It will be sufficient to lay off the distances corresponding to the numbers from 1 to 10, the logarithms being taken from any table.

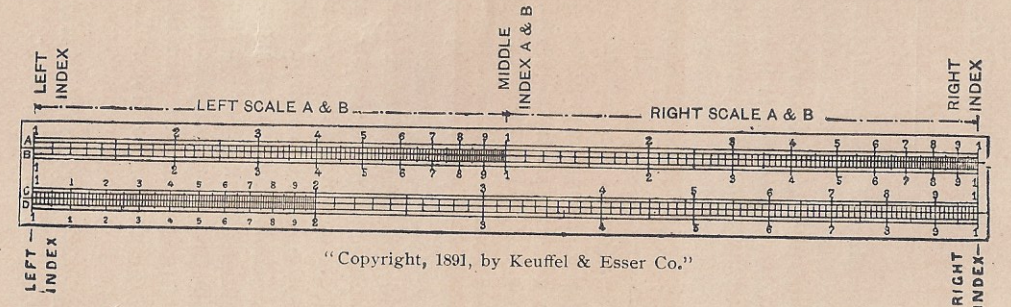
The explanation applies particularly to the TAVERNIER-GRAVET rule, but the principles set forth are general.

For the illustrations on the next page, I am indebted to KEUFFEL & ESSER CO., of New York City, by whom they are copyrighted.

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Troy, N. Y., May, 1891.



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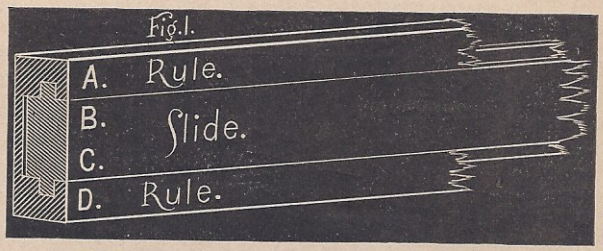


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KEUFFEL & ESSER CO.'S MANNHEIM SLIDE RULE.

THE SLIDE RULE.

1. Description of the Rule.—The Mannheim Slide Rule consists of a frame-work called the *Rule* and a movable part called the *Slide*, arranged as shown in Fig. 1. On their surfaces,



which should be in the same plane, are scales A, B, C, D; A and B are constructed in the same way as the scale B (Art. 2) while C and D are similar to the scale C. The initial points of these scales are in a line perpendicular to the upper edge of the rule. A *Runner*, acting on the principle of a T-square, assists in finding points on the scales that are at a common distance from the initial points of the scales.

The slide bears on its other side a scale of logarithmic sines, one of logarithmic tangents, and a scale of equal parts. The slide is so constructed that it may be inverted,—turned end for end so that B is adjacent to D and C to A,—or reversed so that the opposite side of the slide becomes visible.

2. Construction of the Scales.—A *logarithmic scale* is one on which the distance from the initial point to any division is proportional to the *mantissa* of the logarithm of the number corresponding to that division. The slide rule bears two of these scales, constructed as follows:

On scale B, let a distance of 5 inches (approximately) represent unity in the logarithm, so that, if $a'j' = 5''$, we have

log 1 = 0.00 ; and the beginning of the scale is marked 1.

log 2 = 0.30 ; and at b', so that $a'b' = 1.5''$, we mark 2.

log 3 = 0.48 ; and at c', so that $a'c' = 2.4''$, we. mark 3.

log 4 = 0.60 ; and at d', so that $a'd' = 3.0''$, we mark 4.

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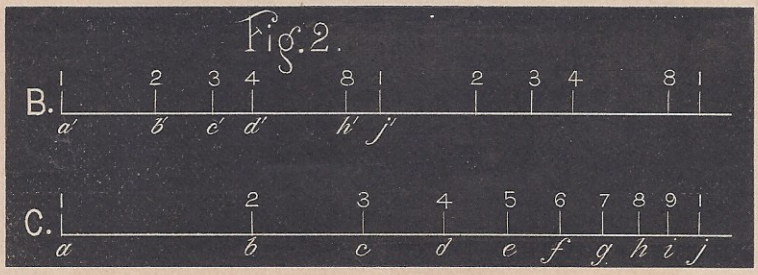
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log 8 = 0.90 ; and at h', so that $a'h' = 4.5''$, we mark 8.

log 10 = 1.00 ; and at j', so that $a'j' = 5.0''$, we mark 1.

This is duplicated on the right of j' so that the total length of scale B is $10''$ (approximately).

Fig. 2.



On scale C, let a distance of 10 inches (approximately) represent unity in the logarithm, so that, if $aj = 10''$, we have

log 1 = 0.00 ; and the beginning of the scale is marked 1.

log 2 = 0.30 ; and at b, so that $ab = 3.0''$, we mark 2.

log 3 = 0.48 ; and at c, so that $ac = 4.8''$, we mark 3.

log 4 = 0.60 ; and at d, so that $ad = 6.0''$, we mark 4.

. . . . . .

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log 8 = 0.90 ; and at h, so that $ah = 9.0''$, we mark 8.

log 9 = 0.95 ; and at i, so that $ai = 9.5''$, we mark 9.

log 10 = 1.00 ; and at j, so that $aj = 10.0''$, we mark 1.

The initial points, a and a', of these scales are in the same vertical line, and aj is exactly twice the length of $a'j'$.

3. Use of the Scales.—Suppose we wish to find the product of 2 and 3, using scale *C*. The logarithm of the product is equal to the sum of the logarithms of the two numbers. Then, if with a pair of dividers we lay off $r\,s = a\,b = \log 2$ and

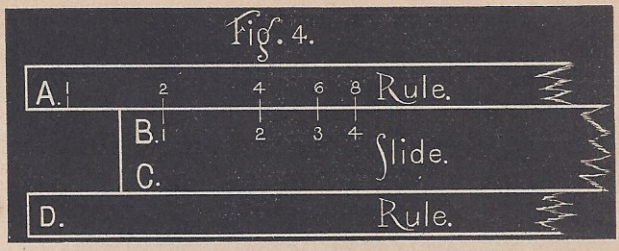


$s\,t = a\,c = \log 3$, $r\,t$ will represent the logarithm of the product and we find by comparison with scale *C* that $r\,t = a\,f = \log 6$.

To divide 8 by 2 we must subtract log 2 from log 8. Lay off with the dividers $r\,t = a\,h = \log 8$ and $t\,s = a\,b = \log 2$. Then $r\,s$ will represent the logarithm of the quotient, and by comparison with scale *C* we find that $r\,s = a\,d = \log 4$.

4. Use of the Rule.—

1. To find the product of two numbers, say 2×3, with the rule, set 1 on scale *B* opposite 2 on scale *A* and opposite 3 on scale *B* find the product 6 on scale *A*. For in this way to log 2 on scale *A* we add log 3, found on scale *B*, giving log 6 on scale *A*.



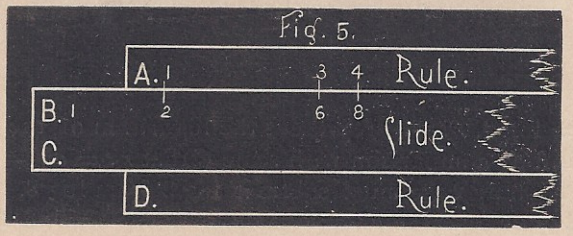
If 2 is to be multiplied by any other number, as 4, the same setting is made and opposite 4 on *B* we find the product 8 on *A*.

If a number a is to be multiplied by a series of numbers, so that we wish to find the value of $a\,x$ for different values of x, opposite a on *A* set 1 on *B* and opposite x on *B* read the product $a\,x$ on *A*. This operation may be stated as follows:

A.	Opp. a.	Read $a\,x$.
B.	Set 1.	Opp. x.

2. To divide a number by another, as $8 \div 2$, set 2 on B opposite 1 on A and opposite 8 on B read the quotient 4 on A. For in this way we take away log 2 from log 8 on scale B and then find the number on A that corresponds to the remainder.

Fig. 5.



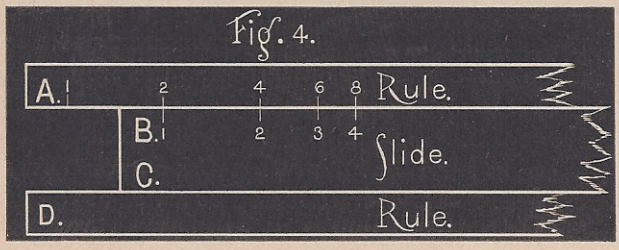
To divide 6 by 2 we could use the same setting, reading the quotient on A opposite 6 on B.

To divide a series of numbers, x, by a constant, b, opposite 1 on A set b on B and opposite x on B read the quotient $x \div b$ on A.

A.	Opp. 1.	Read $x \div b$.
B.	Set b.	Opp. x.

3. A proportion, or an expression in the form $\frac{a\,x}{b}$, as $\frac{4 \times 3}{2}$, may be written $\frac{4}{2} \times 3$. Opposite 4 on A set 2 on B and the index (1) on B will be opposite the quotient $4 \div 2$ on A.

Fig. 4.



Then opposite 3 on B read the result on A, for in this way we add log 3 to the logarithm of the quotient.

If a and b are constants and x a variable we may use the following setting for $\frac{a\,x}{b}$:

A.	Opp. a.	Read $a\,x \div b$.
B.	Set b.	Opp. x.

It will be seen that $\frac{a\,x}{b}$ becomes $a\,x$ when $b = 1$ and $\frac{x}{b}$ when $a = 1$. Compare the settings given for multiplication and division with that for proportion.

5. **Infinite Extent of the Logarithmic Scale.**—In the common system of logarithms a single mantissa will correspond to a given sequence of figures, the position of the decimal point affecting only the characteristic, and to a given mantissa a single sequence of figures will correspond, the position of the decimal point being fixed by the characteristic. Thus

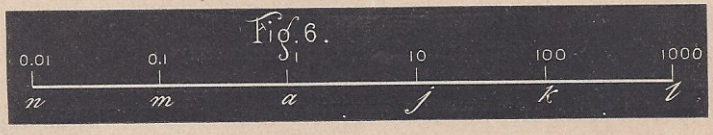
$$\log\ 3 = 0.48\,;$$

$$\log\ 30 = 1.48\,;$$

$$\log\ 300 = 2.48.$$

If we make $a\,j = j\,k = k\,l = n\,m = m\,a$ and if $a\,j = \log 10$, then

Fig. 6.



$$a\,k = 2 \log 10 = \log 10^2 = \log\ 100\,;$$

$$a\,l = 3 \log 10 = \log 10^3 = \log 1000\,;$$

$$a\,m = -\log 10 = \log 10^{-1} = \log \frac{1}{10} = \log 0.1\,;$$

$$a\,n = -2 \log 10 = \log 10^{-2} = \log \frac{1}{10^2} = \log 0.01.$$

Hence the logarithm of a number between 10 and 100 will be represented by a distance greater than $a\,j$ and less than $a\,k$. The logarithmic scale, therefore, is not merely the distance $a\,j$ but it is composed of a series of such parts, extending without

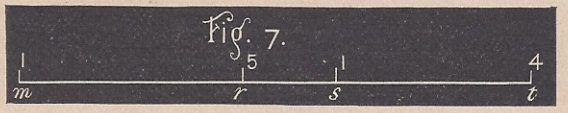
limit on both sides of a. Any one of the points n, m, a, j, k, l, and so on, may be considered as the starting point ($\log 1 = 0$); thus, if the scale is supposed to start at n, we would mark 1 at n, 10 at m, 100 at a, 1000 at j, and so on. For this reason the indices of scales B and C are marked 1 and not with multiples of 10.

To illustrate this, suppose we wish to multiply 5 by 8. In Fig. 3, using scale C, make $r\,s = a\,e = \log 5$ and $s\,t = a\,h = \log 8$. Then comparing the distance $r\,t$ with scale C we find that $r\,t$ is greater than $a\,j$, the excess being equal to $a\,d = \log 4$. Hence

$$\log (5 \times 8) = \log 5 + \log 8 = a\,j + a\,d = \log 10 + \log 4 = \log 40.$$

Again, suppose we wish to divide 4 by 8. Make $s\,t = \log 4$ and $t\,r = \log 8$. Then $s\,r$ is a *negative* mantissa, and

$$-\,r\,s = -\,m\,s + m\,r = -\,1 + m\,r.$$



By comparison with scale C we find that $m\,r$ is the mantissa of $\log 5$, so that the quotient is 0.5.

6. Sequence of Figures.—In operations with the slide rule we use the mantissas of the logarithms of the quantities involved and the distances found represent the mantissas of the results. We find, therefore, the sequence of figures in the result, the position of the decimal point being determined either by special rules or by rough computation.

7. Shifting the Slide.—As we use the mantissas only and as the addition or subtraction of the distance between the indices of the scales would affect the characteristic alone, we can change the position of the slide by bringing one index into the position previously occupied by another index without affecting the sequence of figures in the result.

To illustrate the use of the runner and the principle involved in the shifting of the slide, let us find the value of $2 \times 3 \times 4$:

1. Using A and B. Opposite 2 on A set 1 on B and 3 on B will be opposite 6 ($= 2 \times 3$) on A. Instead of reading this number, set the runner over 3 on B. Bring 1 on B to the runner and now read the final result, 24, on A opposite 4 on B.

2. Using C and D. Opposite 2 on D set 1 on C and 3 on C will be opposite 6 on D. Set the runner over 3 on C without reading the 6. Bring 1 on C to the runner and opposite 4 on C read the result, 24, on D. But if we have used the left index (1) of C, we shall find that 4 on C is outside the rule, so that we cannot read the result. To overcome this, bring the *right* index of C to the runner, thus shifting the slide over the entire length of its scales, and the result can be read.

8. Squares and Square Roots.—The distance on C representing log 2 ($= 3.0''$) is just twice that representing log 2 ($= 1.5''$) on B. Hence, if x is any number, log x on C is twice log x on B. If the initial points of the two scales are in the same vertical line any number, x, on C will correspond to its square, x^2, on B; for

$$\log x \text{ on } C = 2 \log x \text{ on } B = \log x^2 \text{ on } B.$$

Conversely, any number on B will correspond to its square root on C.

1. Squares. To find the square of any number, find the number on D (or C) and the corresponding number on A (or B) will be its square.

2. Square roots. To find the square root of any number, find the number on A (or B) and the corresponding number on D (or C) will be its square root. To avoid artificial rules it is necessary to know approximately the first figure of the root. Divide the number into sections of two digits each, commencing at the decimal point as in arithmetic, and call the first section on the left that contains a significant figure the *left section*. If this left section is 10 or greater the first figure of the root will be 3 or greater, while if it is less than 10 the first figure of the root will be 3 or less. In the Mannheim rule the middle index of A (or B) corresponds to 3.16 + on D (or C) since $\sqrt{10} = 3.1622+$. Therefore if the left section is less than 10 the number is found on the left scale of A (or B), and if greater than 10 the number is found on the right scale of A (or B)*.

9. The Arithmetical Complement.—The arithmetical complement (written *colog*) of a number is the logarithm of its

*Or the number is found on the left scale of A (or B) when there is an odd number of digits to the left or an odd number of ciphers to the right of the decimal point; otherwise it is found on the right scale of A (or B).

reciprocal. The mantissa of the cologarithm of a number is found by subtracting the mantissa of the logarithm from unity. Thus

mantissa of log 2 = .301030
" " log 0.5 = .698970
∴ sum of mantissas = 1.000000

Hence, if in Fig. 2 ab represents log 2 on C, bj will represent colog 2, since $bj = aj - ab = \log 10 - \log 2 = 1 - \log 2$.

10. Principles of Logarithms.—

1. To a given mantissa in the common system of logarithms a single sequence of figures will correspond, the position of the decimal point being determined by the characteristic; and to a given sequence of figures there will correspond a single mantissa, the characteristic depending upon the position of the decimal point.

2. The arithmetical complement of the logarithm (written *colog* or *col*) of a number is the logarithm of its reciprocal. The sum of the mantissas of the logarithm and of the cologarithm of a number is always equal to unity.

3. To find the logarithm of the product of two numbers:
(*a*). Add their logarithms.
(*b*). Subtract the cologarithm of one from the logarithm of the other.

4. To find the logarithm of the quotient of two numbers:
(*a*). Subtract the logarithm of the divisor from the logarithm of the dividend.
(*b*). Add the cologarithm of the divisor to the logarithm of the dividend.

5. To find the logarithm of a power of a number, multiply the logarithm of the number by the index of the power.

6. To find the logarithm of a root of a number, divide the logarithm of the number by the index of the root.

11. Principles of the Slide Rule.—

1. A logarithmic scale is one on which the distance from the initial point to any division is proportional to the *mantissa* of the logarithm of the number corresponding to that division. The scales A, B, C, D, are logarithmic.

2. The distance from the initial point to a number x on any

scale represents the mantissa of log x on that scale, while the distance from x to the following index (1) represents the mantissa of colog x.

3. When the slide is inverted (turned end for end) the distance from the left index to any number x represents the mantissa of colog x while the distance from x to the following index (1) represents the mantissa of log x.

4. In operations with the slide rule we use the mantissas of the logarithms of the quantities involved and the distances found represent the mantissas of the logarithms of the results. We find, therefore, the sequence of figures in the result, the position of the decimal point being determined either by special rules or by rough computation.

5. We may shift the slide over the entire length of its scales without affecting the sequence of figures in the result; for the distance between two consecutive indices represents log 10 (= 1), so that the shifting would affect the characteristic only.

6. Since the distance on D representing log 2 is twice that representing log 2 on A, we have

$$\log 2 \text{ on } D = 2 \log 2 \text{ on } A = \log 2^2 \text{ on } A.$$

7. If a number is squared, find the number on D (or C) and the corresponding number on A (or B) will be its square.

8. If the square root of a number is required, find the number on A (or B) and the corresponding number on D (or C) will be its square root.

9. In continued operations, as in finding the product of three numbers, the results should be found on the rule in order that the runner may be used.

10. Since log 10 on D is twice the length of log 10 on A, the result should be read on D rather than on A, other things being equal.

12. General Statement of Problems.—The value of any expression in a fractional form with two factors in the numerator and one in the denominator, one of the numbers being variable and any one or all of the numbers being squared, and also the value of the square root of such an expression, may be found with one setting of the Mannheim rule, the result being read on the rule.

Let a denote one of the factors in the numerator, x the

variable number, b the third number, and y the value of the expression. If x is in the numerator b will be in the denominator, while if x is in the denominator b will be in the numerator. There are 26 general forms and 34 special cases:

Genl.	Special.	Genl.	Special.	Genl.	Special.	Genl.	Special.
$\frac{a\,x}{b}$.	$a\,x$, $\frac{x}{b}$.	$\sqrt{\frac{a\,x}{b}}$.	$\sqrt{a\,x}$, $\sqrt{\frac{x}{b}}$.	$\frac{a\,b}{x}$.	$\frac{a}{x}$, $\frac{1}{x}$.	$\sqrt{\frac{a\,b}{x}}$.	$\sqrt{\frac{a}{x}}$, $\sqrt{\frac{1}{x}}$.
$\frac{a^2\,x}{b}$.	$a^2\,x$, a^3.	$\sqrt{\frac{a^2\,x}{b}}$.	$\sqrt{a^2\,x}$, $\sqrt{a^3}$.	$\frac{a^2\,b}{x}$.	$\frac{a^2}{x}$, $\frac{a^3}{x}$.	$\sqrt{\frac{a^2\,b}{x}}$.	$\sqrt{\frac{a^2}{x}}$, $\sqrt{\frac{a^3}{x}}$.
$\frac{a\,x}{b^2}$.	$\frac{x}{b^2}$.	$\sqrt{\frac{a\,x}{b^2}}$.	$\sqrt{\frac{x}{b^2}}$.	$\frac{a^2\,b^2}{x}$.	$\frac{a^4}{x}$.	$\sqrt{\frac{a^2\,b^2}{x}}$.	$\sqrt{\frac{a^4}{x}}$.
$\frac{a^2\,x}{b^2}$.		$\sqrt{\frac{a^2\,x}{b^2}}$.		$\frac{a\,b}{x^2}$.	$\frac{a}{x^2}$, $\frac{1}{x^2}$.	$\sqrt{\frac{a\,b}{x^2}}$.	$\sqrt{\frac{a}{x^2}}$.
$\frac{a\,x^2}{b}$.	$a\,x^2$, $\frac{x^2}{b}$.	$\sqrt{\frac{a\,x^2}{b}}$.	$\sqrt{a\,x^2}$, $\sqrt{\frac{x^2}{b}}$.	$\frac{a^2\,b}{x^2}$.	$\frac{a^2}{x^2}$, $\frac{a^3}{x^2}$.	$\sqrt{\frac{a^2\,b}{x^2}}$.	$\sqrt{\frac{a^3}{x^2}}$.
$\frac{a^2\,x^2}{b}$.	$a^2\,x^2$, a^4.	$\sqrt{\frac{a^2\,x^2}{b}}$.		$\frac{a^2\,b^2}{x^2}$.	$\frac{a^4}{x^2}$.		
$\frac{a\,x^2}{b^2}$.	$\frac{x^2}{b^2}$.	$\sqrt{\frac{a\,x^2}{b^2}}$.					
$\frac{a^2\,x^2}{b^2}$.							

13. General Method.—The following statements will give the setting for any one of these cases:

1. If x is in the numerator, use the slide direct.
   If x is in the denominator, use the slide inverted.
2. Find a and y on the rule.
   Find b and x on the slide.
3. Find a on A if first power, on D if squared.
   Find b on B if first power, on C if squared.
   Find x on B if first power, on C if squared.
   Find y on A if first power, on D if it is a square root.

For illustration take

$$y = \frac{a x^2}{b^2}.$$

$$\log y = (\log a - \log b^2) + \log x^2;$$

x is in the numerator, therefore use the slide direct.
a is first power; $\therefore$ opposite a on A, since
b is second power, set b on C;
x is second power; $\therefore$ opposite x on C, since
y is first power, read y on A.

$$y = \sqrt{\frac{a^2 b}{x^2}}.$$

$$\log y = \tfrac{1}{2}[(\log a^2 - \operatorname{colog} b) + \operatorname{colog} x^2];$$

x is in the denominator, therefore use the slide inverted.
a is second power; $\therefore$ opposite a on D, since
b is first power, set b on B;
x is second power; $\therefore$ opposite x on C, since
y is a square root, read y on D.

The caution given in Art. 8 must be carefully heeded in finding a square root.

14. Settings for the General Cases.—

$$y = \frac{a x}{b}.$$

$\log y = (\log a - \log b) + \log x.$

A.	Opp. a.	Read y.
B.	Set b.	Opp. x.
C.		
D.		

Ex. $a = 3, b = 2$; $x = 4, y = 6$; $x = 6, y = 9$; $x = 8, y = 12$.

$$y = \frac{a^2 x}{b}.$$

$\log y = (\log a^2 - \log b) + \log x.$

A.		Read y.
B.	Set b.	Opp. x.
C.		
D.	Opp. a.	

Ex. $a = 3, b = 2$; $x = 4, y = 18$; $x = 6, y = 27$; $x = 8, y = 36$.

$$y = \frac{a\,x}{b^2}.$$

$\log y = (\log a - \log b^2) + \log x.$

A. Opp. *a.*	Read *y*.
B.	Opp. *x*.
C. Set *b*.	
D.	

Ex. $a = 6, b = 3$; $x = 2, y = 1.33$;
$x = 4, y = 2.67$;
$x = 8, y = 5.33$.

$$y = \frac{a^2\,x}{b^2}.$$

$\log y = (\log a^2 - \log b^2) + \log x.$

A.	Read *y*.
B.	Opp. *x*.
C. Set *b*.	
D. Opp. *a*.	

Ex. $a = 3, b = 2$; $x = 2, y = 4.5$;
$x = 4, y = 9.0$;
$x = 8, y = 18.0$.

$$y = \frac{a\,x^2}{b}.$$

$\log y = (\log a - \log b) + \log x^2.$

A. Opp. *a*.	Read *y*.
B. Set *b*.	
C.	Opp. *x*.
D.	

Ex. $a = 3, b = 2$; $x = 4, y = 24$;
$x = 6, y = 54$;
$x = 8, y = 96$.

$$y = \frac{a^2\,x^2}{b}.$$

$\log y = (\log a^2 - \log b) + \log x^2.$

A.	Read *y*.
B. Set *b*.	
C.	Opp. *x*.
D. Opp. *a*.	

Ex. $a = 2, b = 3$; $x = 2, y = 5.33$;
$x = 3, y = 12.00$;
$x = 9, y = 108.00$.

$$y = \frac{a\,x^2}{b^2}.$$

$\log y = (\log a - \log b^2) + \log x^2.$

A. Opp. *a*.	Read *y*.
B.	
C. Set *b*.	Opp. *x*.
D.	

Ex. $a = 3, b = 2$; $x = 3, y = 6.75$;
$x = 6, y = 27.00$;
$x = 8, y = 48.00$.

$$y = \frac{a^2\,x^2}{b^2}.$$

$\log y = (\log a^2 - \log b^2) + \log x^2$

A.	Read *y*.
B.	
C. Set *b*.	Opp. *x*.
D. Opp. *a*.	

Ex. $a = 2, b = 4$; $x = 3, y = 2.25$;
$x = 6, y = 9.00$;
$x = 8, y = 16.00$.

$$y = \frac{a\,b}{x}.$$

$\log y = (\log a - \operatorname{colog} b) + \operatorname{colog} x.$

	A. Opp. *a.*	Read *y.*
(Inv.)	*C.*	
	B. Set *b.*	Opp. *x.*
	D.	

Ex. $a = 5, b = 12$; $x = 2, y = 30.0$; $x = 4, y = 15.0$; $x = 8, y = 7.5$.

$$y = \frac{a^2\,b}{x}.$$

$\log y = (\log a^2 - \operatorname{colog} b) + \operatorname{colog} x.$

	A.	Read *y.*
(Inv.)	*C.*	
	B. Set *b.*	Opp. *x.*
	D. Opp. *a.*	

Ex. $a = 2, b = 15$; $x = 3, y = 20.0$; $x = 5, y = 12.0$; $x = 8, y = 7.5$.

$$y = \frac{a^2\,b^2}{x}.$$

$\log y = (\log a^2 - \operatorname{colog} b^2) + \operatorname{colog} x.$

	A.	Read *y.*
(Inv.)	*C.* Set *b.*	
	B.	Opp. *x.*
	D. Opp. *a.*	

Ex. $a = 2, b = 6$; $x = 2, y = 72$; $x = 4, y = 36$; $x = 8, y = 18$.

$$y = \frac{a\,b}{x^2}.$$

$\log y = (\log a - \operatorname{colog} b) + \operatorname{colog} x^2.$

	A. Opp. *a.*	Read *y.*
(Inv.)	*C.*	Opp. *x.*
	B. Set *b.*	
	D.	

Ex. $a = 5, b = 12$; $x = 2, y = 15.00$; $x = 4, y = 3.75$; $x = 5, y = 2.4$.

$$y = \frac{a^2\,b}{x^2}.$$

$\log y = (\log a^2 - \operatorname{colog} b) + \operatorname{colog} x^2.$

	A.	Read *y.*
(Inv.)	*C.*	Opp. *x.*
	B. Set *b.*	
	D. Opp. *a.*	

Ex. $a = 2, b = 15$; $x = 3, y = 6.67$; $x = 4, y = 3.75$; $x = 5, y = 2.4$.

$$y = \frac{a^2\,b^2}{x^2}.$$

$\log y = (\log a^2 - \operatorname{colog} b^2) + \operatorname{colog} x^2.$

	A.	Read *y.*
(Inv.)	*C.* Set *b.*	Opp. *x.*
	B.	
	D. Opp. *a.*	

Ex. $a = 2, b = 6$; $x = 3, y = 16.00$; $x = 4, y = 9.00$; $x = 8, y = 2.25$.

15. Square Roots of the General Expressions.—Set the rule as in finding the value of the expression under the radical sign, but read the result on D instead of on A. Since any sequence of figures will have two square roots, their values depending upon the position of the decimal point, the wrong root may be taken if proper care is not exercised.

16. Use of Scales C and D.—Some of the expressions may be solved with one setting using only the scales C and D; thus,

$$y = \frac{a\,x}{b}.$$

C.	Set b.	Opp. x.
D.	Opp. a.	Read y.

$$y = \frac{a\,b}{x}.$$

(Inv.)	C.	Set b.	Opp. x.
	D.	Opp. a.	Read y.

Others may be solved using C and D with the runner by making more than one setting; thus, R denoting the runner,

$$y = \frac{a^2\,x}{b^2} = \frac{a^2}{b}\,\frac{x}{b}.$$

C.	Set b.	R to a.	b to R.	Opp. x.
D.	Opp. a.			Read y.

17. Use of the Runner in Complicated Expressions.—If we have found the value of an expression, say $\frac{a\,b}{c}$, and marked its place on the rule by placing the runner there, then to multiply it by a fraction $\frac{d}{e}$ we bring the denominator e on the slide to the runner and read the result on the rule opposite the numerator d on the slide. For this first subtracts log e from log $\frac{a\,b}{c}$ and then adds log d to the remainder.

$$y = \frac{a\,b\,c\,d\,e}{f\,g\,h\,k} = \frac{a\,b}{f} \cdot \frac{c}{g} \cdot \frac{d}{h} \cdot \frac{e}{k}.$$

1.

A.	Opp. *a.*	Read *y.*
B.	Set *f.* *R* to *b.* *g* to *R.* *R* to *c.* *h* to *R.* *R* to *d.* *k* to *R.*	Opp. *e.*

2.

C.	Set *f.* *R* to *b.* *g* to *R.* *R* to *c.* *h* to *R.* *R* to *d.* *k* to *R.*	Opp. *e.*
D.	Opp. *a.*	Read *y.*

The solution with *C* and *D* will usually require the shifting of the slide over the length of its scales.

Examples:

$$\frac{4.\ 5.\ 6.\ 7}{2.\ 3.\ 4} = \frac{4.\ 5}{2} \cdot \frac{6}{3} \cdot \frac{7}{4} = 35.$$

$$\frac{5.\ 6.\ 7.\ 8}{2.\ 4.\ 5.\ 6} = \frac{5.\ 6}{2} \cdot \frac{7}{4} \cdot \frac{8}{5} \cdot \frac{1}{6} = 7.$$

$$\frac{8.\ 9}{2.\ 3.\ 4.\ 6} = \frac{8.\ 9}{2} \cdot \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{1}{6} = 0.5.$$

When an expression contains a square root multiplied by factors with integral exponents, the value of the quantity under the radical may be found on *A* and the square root marked on *D* with the runner, the multiplication of this quantity by the integral factors being effected by the use of *C* and *D*. Thus,

$$y = \frac{a^2\, b^{\frac{1}{2}}}{c} \sqrt{\frac{e}{f\,g}} = \frac{a^2}{c} \sqrt{\frac{b\,e}{f\,g}} = \sqrt{\frac{b\,e}{f} \cdot \frac{1}{g}} \times \frac{a}{c} \cdot \frac{a}{1}.$$

A.	Opp. *b.*	
B.	Set *f.* *R* to *e.* *g* to *R.* *R* to 1.	
C.		*c* to *R.* *R* to *a.* 1 to *R.* Opp. *a.*
D.		Read *y.*

The following setting depends upon the fact that, if we wish to multiply a number on *D* by the square root of a number, we must find on *B* the number whose root is used. Thus,

$$y = \frac{a^2 b^{\frac{3}{2}}}{c\,d}\sqrt{\frac{e}{f\,g}} = \frac{a}{c}\cdot\frac{a}{d}\cdot\frac{b}{d}\cdot\sqrt{\frac{b}{f}}\cdot\sqrt{\frac{e}{g}}.$$

A.	
B.	f to R. R to b. g to R. Opp. e.
C. Set c. R to a. d to R. R to b.	
D. Opp. a.	Read y.

An expression of the form

$$y = \frac{a\,(m^2 + n^2 + r + s\,p + q^3)}{b}$$

may be written

$$y = \frac{a}{b}m^2 + \frac{a}{b}n^2 + \frac{a}{b}r + \frac{a}{b}s\,p + \frac{a}{b}q^3$$

and the values of the several terms found separately. Notice that one setting will serve for the first three terms.

18. Gauge-points.—When a formula is often used, as that for computing the horse-power of an engine, it will be found convenient to combine the constant factors, sometimes using the resulting constant and sometimes its reciprocal. Such a constant is called a *gauge-point*. Thus the formula for the horse-power of an engine* is

$$H.\,P. = \frac{p \times 0.7854\,d^2 \times 2\,s \times r}{33000}$$

where p = mean effective steam pressure in the cylinder in lbs. per sq. in.,

d = diameter of piston in inches,

s = stroke in feet,

r = number of revolutions per minute.

But $\dfrac{0.7854 \times 2}{33000} = \dfrac{1}{21008}$; $\therefore H.\,P. = \dfrac{p\,d^2\,s\,r}{21008}$

where 21008 is the gauge-point for the formula.

* This example was given by Mr. Wm. Cox in *Engineering News*, Dec. 20, 1890.

$$H.\ P. = \frac{p\,d^2}{21008} \times s \times r.$$

A.	Opp. *p.*					Read *H. P.*
B.	Set 21008.		1 to *R.*	*R* to *s.*	1 to *R.*	Opp. *r.*
C.		*R* to *d.*				
D.						

$$H.\ P. = \frac{p\,d}{21008} \times d \times s \times r.$$

C.	Set 21008.	*R* to *d.*	1 to *R.*	*R* to *d.*	1 to *R.*	*R* to *s.* 1 to *R.* Opp. *r.*
D.	Opp. *p.*					Read *H. P.*

The gauge-points used in finding the weights of prisms, cylinders, and spheres, of certain substances are given on the back of the rule. The values on French and German rules are based upon the metric system in which the weight of a cubic centimeter of water is one gramme, and the dimensions of the solid are supposed to be measured in centimeters. If w represents the specific gravity of the solid, we have the following formulæ :

For prisms; vol. $= a\,b\,c$; weight $= w\,a\,b\,c \quad = a\,b\,c \div \frac{1}{w}$.

For cylinders; vol. $= \frac{\pi\,d^2\,h}{4}$; weight $= w\,\frac{\pi\,d^2\,h}{4} = d^2\,h \div \frac{4}{\pi\,w}$.

For spheres; vol. $= \frac{\pi\,d^3}{6}$; weight $= w\,\frac{\pi\,d^3}{6} \quad = d^3 \quad \div \frac{6}{\pi\,w}$.

For water, $w = 1$;

$\therefore$ for prisms, the gauge-point $= \frac{1}{w} \quad = 1$;

for cylinders, the gauge-point $= \frac{4}{\pi\,w} = 1.273$;

for spheres, the gauge-point $= \frac{6}{\pi\,w} = 1.910$.

For zinc, $w = 7.2$ approximately ;

$\therefore$ for prisms, the gauge-point $= \frac{1}{w} = 0.139$;

for cylinders, the gauge-point $= \frac{4}{\pi w} = 0.177$;

for spheres, the gauge-point $= \frac{6}{\pi w} = 0.265$.

The gauge-points would be different for English measures. The "Carpenter's Rule" has on it columns marked $F\,F\,F$, $F\,I\,I$, $I\,I\,I$, for prisms, $F\,I$ and $I\,I$ for cylinders, and F and I for spheres, F and I denoting the dimensions measured in feet and inches respectively. Suppose we wish to find the gauge-point for a cylinder of zinc ($w = 7.2$), the diameter being in inches and the height in feet ($F\,I$).

$$\therefore \text{volume} = \frac{\pi d^2 h}{4 \times 144} \text{ cu. ft.};$$

and if the weight of a cubic foot of water is 62.5 pounds,

$$\text{weight} = w \times 62.5 \times \frac{\pi d^2 h}{4 \times 144} = d^2 h \div \frac{4 \times 144}{\pi w \times 62.5}$$

$$= d^2 h \div 0.407 ;$$

so that the gauge-point would be 0.407.

19. Extraction of Cube Roots, $y = \sqrt[3]{a}$.—Since $\sqrt[3]{8.000} = 2$, $\sqrt[3]{80.000} = 4 +$, $\sqrt[3]{800.000} = 9 +$, the cube root of any sequence of figures will depend upon the position of the decimal point, so that the first figure of the root should be known approximately in order that the wrong number may not be taken. This first figure may be found by dividing the given number into sections containing three digits each, commencing at the decimal point, and extracting the cube root of the left section. Then, with the slide *inverted*, opposite a on A set 1 on C and find

the number on D that corresponds to the same number on B. This will be the required cube root, for

$$\log a = 2 \log y - \operatorname{colog} y = 2 \log y + \log y.$$

	A.	Opp. a.	
(Inv.)	C.	Set 1.	
	B.		Opp. y.
	D.		Read y.

20. Extraction of Fourth Roots, $y = \sqrt[4]{a}$.—The method involves the extraction of the square root of the square root of a. Since $\sqrt[4]{6.0000} = 1+$, $\sqrt[4]{60.0000} = 2+$, $\sqrt[4]{600.0000} = 4+$, and $\sqrt[4]{6000.0000} = 8+$, we see that any sequence of figures has four fourth roots, the true root being determined by the position of the decimal point.

Find the first figure of the *square* root of a approximately. Using the slide *inverted*, set the index (1) of C opposite a on A, using that scale of A on which a is approximately opposite the first figure of the square root on D. The fourth root of a will be the number on D that is opposite the same number on C. Use the right index of C when in $\sqrt{a}$ there would be an odd number of digits to the left or an odd number of ciphers to the right of the decimal point; otherwise use the left index of C.

Let $y' = \sqrt{a}$. We first find $\log y'$ on D, and then

1. With an odd number of digits or ciphers in $\sqrt{a}$,

$$\log y' = 2 \log y = \log y + \log y.$$

2. With an even number of digits or ciphers in $\sqrt{a}$,

$$\log y' = 2 \log y = \log y - \operatorname{colog} y.$$

	A.	Opp. a.	
(Inv.)	C.	Set 1.	Opp. y.
	B.		
	D.		Read y.

Compare this setting with that of Art. 19.

SLIDE REVERSED.

21. The Reverse of the Slide has a scale of logarithmic sines along one edge and one of logarithmic tangents along the other, with a scale of equal parts in the center. The scales of sines and tangents are so arranged that the numbers (degrees) will increase from left to right when, the slide being reversed, the corresponding edge of the slide is adjacent to A.

22. The Scale of Logarithmic Sines, S.—If we reverse the slide and place S adjacent to A so that their initial points coincide, we find that the beginning of A corresponds to $0° 34' +$ on S, the middle index of A to $5° 44' +$, and the end of A to $90°$; for

$$\log \sin \ 0° 34' + = 8.00 (= -2),$$
$$\log \sin \ 5° 44' + = 9.00 (= -1),$$
$$\log \sin 90° \qquad = 0.00,$$

unity in the characteristic being represented by the same distance *on the sine scale and on A*.

In this position the natural sine of an angle may be found by reading the number on A corresponding to the angle on S.

The distance from the beginning of S to any division, reading in the direction of increasing numbers, represents the mantissa, or the mantissa + 1, of the logarithmic sine of the angle corresponding to that division; and the distance from any division to the other end (90°) of S represents the mantissa, or the mantissa + 1, of the cologarithm of the sine.

In expressions containing $\cos x$, since $\cos x = \sin (90° - x)$, we may subtract the given angle from 90° and then use the sine of the remainder, found on the sine scale.

1. $a = c \sin x$.

$\therefore \log a = \log c + \log \sin x = \log c - \operatorname{colog} \sin x$.

Opposite c on A set the beginning of S and opposite x on S read a on A, for this adds $\log \sin x$ to $\log c$. If x should fall outside of the rule we would set the *right* end of S opposite c on A and then opposite x on S read a on A, for this subtracts $\operatorname{colog} \sin x$ from $\log c$.

2. $b = c \cos x$.

$\therefore \log b = \log c + \log \sin (90° - x) = \log c - \operatorname{colog} \sin (90° - x)$,

and the methods given for the multiplication of a sine by a number ($a = c \sin x$) are applicable, using $90° - x$ instead of x.

3. $c = a \div \sin x$.

$\therefore \log c = \log a - \log \sin x = \log a + \operatorname{colog} \sin x$.

Opposite a on A set x on S and opposite the end of S read c on A; for this subtracts log sin x from log a when we read opposite the left end and adds colog sin x to log a when we read opposite the right end.

4. $c = b \div \cos x$.

$\therefore \log c = \log b - \log \sin (90° - x) = \log b + \operatorname{colog} \sin (90° - x)$,

and the methods for $c = a \div \sin x$ are applicable, using $90° - x$ instead of x.

5. $a = c \sin x \div \sin z$.

$\therefore \log a = \log c - \log \sin z + \log \sin x$.

Opposite c on A set z on S and opposite x on S read a on A; for this first subtracts log sin z from log c and then adds log sin x to the remainder. If x falls outside the rule, set the runner over the end of S that is in the rule, shift the slide until the other end of S comes to the runner, and then opposite x on S read a on A.*.

The expression for a becomes $c \sin x$ when $z = 90°$ and $c \div \sin z$ when $x = 90°$. Compare the setting just given with those for $c \sin x$ and $a \div \sin x$.

6. $\sin x = a \sin z \div c$.

$\therefore \log \sin x = \log \sin z - \log c + \log a$.

Opposite c on A set z on S and opposite a on A read x on S; for this subtracts log c from log sin z and adds log a to the remainder.

This becomes $\sin x = a \div c$ when $z = 90°$.

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*In all cases when the result cannot be read, the slide is shifted over the length of its scales since this merely changes the characteristic.

7. $\cos x = b \div c$.

$$\therefore \log \sin (90° - x) = \log b - \log c$$
$$= \log \sin 90° - \log c + \log b.$$

Opposite c on A set 90° on S and opposite b on A read $90° - x$ on S.

23. The Scale of Logarithmic Tangents, T.—If we reverse the slide and place the tangent scale adjacent to A so that their initial points coincide, we find that the beginning of A corresponds to $0° 34' +$ on T, the middle index of A to $5° 42' +$, and the right index to 45°; for

$$\log \tan \ 0° 34' + = 8.00 \ (= -2),$$
$$\log \tan \ 5° 42' + = 9.00 \ (= -1),$$
$$\log \tan 45° \qquad = 0.00.$$

Unity in the characteristic, therefore, corresponds to the same distance on T, A, and S.

In this position the natural tangent of an angle less than 45° may be found by reading the number on A opposite the given angle on T. To find the tangent of an angle greater than 45°, invert the slide so that T is adjacent to D, their ends coinciding, and read the number on A corresponding to the complement of the angle on T.

The distance from the beginning of T to any division, reading in the direction of increasing numbers, represents the mantissa, or the mantissa + 1, of the logarithmic tangent of the angle corresponding to that division; while the distance from that division to the end (45°) of the scale represents the mantissa, or the mantissa + 1, of the cologarithm of the tangent.

Since $\tan x = \dfrac{1}{\cot x}$, we have

$$\log \tan x = \operatorname{colog} \cot x,$$
$$\text{and } \operatorname{colog} \tan x = \log \cot x.$$

Hence the distance that represents log tan x also represents colog cot x, and that representing colog tan x also represents log cot x.

When x is greater than 45°, subtract x from 90° and use the cotangent of the remainder instead of tan x and the tangent of the remainder instead of cot x.

1. $a = b \tan x$.

$x < 45°$. $\therefore \log a = \log b + \log \tan x = \log b - \operatorname{colog} \tan x$.

Opposite b on A set the end of T and opposite x on T read a on A; for this adds log tan x to log b when the left end is used and subtracts colog tan x from log b when the right end is used.

$x > 45°$. $\therefore \log a = \log b + \log \cot (90° - x)$
$= \log b - \log \tan (90° - x)$
$= \log b + \operatorname{colog} \tan (90° - x)$.

Opposite b on A set $90° - x$ on T and opposite the end of T read a on A; for this subtracts log tan $(90° - x)$ from log b when the left end is used and adds colog tan $(90° - x)$ to log b when the right end is used.

2. $b = a \div \tan x$.

$x < 45°$. $\therefore \log b = \log a - \log \tan x = \log a + \operatorname{colog} \tan x$.

Opposite a on A set x on T and opposite the end of T read b on A; for this subtracts log tan x from, or adds colog tan x to, log a when we read opposite the left or the right end respectively.

$x > 45°$. $\therefore \log b = \log a - \log \cot (90° - x)$
$= \log a + \log \tan (90° - x)$
$= \log a - \operatorname{colog} \tan (90° - x)$.

Opposite a on A set the end of T and opposite $90° - x$ on T read b on A; for this adds log tan $(90° - x)$ to, or subtracts colog tan $(90° - x)$ from, log a when we use the left or the right end respectively.

3. $b = a \cot x$.

$x < 45°$. $\therefore \log b = \log a + \log \cot x$
$= \log a - \log \tan x$
$= \log a + \operatorname{colog} \tan x$.

$x > 45°$. $\therefore \log b = \log a + \log \tan (90° - x)$
$= \log a - \operatorname{colog} \tan (90° - x)$.

The settings given under (2) are used.

4. $a = b \div \cot x$.

$x < 45°$. $\therefore \log a = \log b - \log \cot x$

$= \log b + \log \tan x = \log b - \operatorname{colog} \tan x$.

$x > 45°$. $\therefore \log a = \log b - \log \tan (90° - x)$

$= \log b + \operatorname{colog} \tan (90° - x)$.

The settings given under (1) are used.

24. Angles less than 3°.—On A and B there are two divisions marked with the second and with the minute mark ($''$ and $'$). Their distances from the left index of A or B represent $\log \frac{1}{\sin 1''}$ ($= \operatorname{colog} \sin 1''$) and $\log \frac{1}{\sin 1'}$ ($= \operatorname{colog} \sin 1'$). They may be used to find the sines and tangents of angles less than 3° with an accuracy corresponding to that in the general use of the rule; thus, approximately,

$$\sin x = x'' \sin 1'' = x' \sin 1',$$

$$\tan x = x'' \tan 1'' = x' \tan 1';$$

also $\tan 1'' = \sin 1''$ and $\tan 1' = \sin 1'$ practically.

To find $\sin 1° 10'$ ($= \sin 70'$), we say that $\sin 70' = 70 \sin 1'$.

$\therefore \log \sin 70' = \log 70 + \log \sin 1' = \log 70 - \operatorname{colog} \sin 1'$.

Opposite 70 on A set the minute mark on B and opposite 1 on B read $\sin 1° 10'$ on A ($= 0.0204$).

If $\tan x = 0.0233$ and x is required, we have

$x' \tan 1' = x' \sin 1' = 0.0233$;

$\therefore x' = 0.0233 \div \sin 1'$.

$\therefore \log x' = \log 0.0233 + \operatorname{colog} \sin 1'$.

Opposite the minute mark on A set 1 on B and opposite 233 on B read $x' = 80'$ on A.

25. The Scale of Equal Parts.—This scale, in the center of the reverse of the slide, has the same length as D and is divided into 500 equal parts, so that by comparison with D the logarithm of any number may be found. To facilitate this comparison a portion of the back of the rule is cut away and beveled; with

the slide in its usual position set the left index of C opposite any number on D and opposite the mark on the bevel read the logarithm of that number on the scale of equal parts.

26. To Find Sines and Tangents without Reversing the Slide.—The back of the rule is cut away and beveled at each end and there are two index marks, in addition to the one already mentioned, on the beveled sides. The sine or tangent of an angle may be found without reversing the slide by bringing the degree on the proper scale to the mark opposite that scale, the slide being in its usual position, and then reading the number on B opposite the index of A for a sine or the number on A opposite the index of B for a tangent.

CONCLUSION.

27. Accuracy of the Result.—The errors inherent in the rule are due to warping, to inequality in longitudinal shrinkage, to inaccuracy in the division of the scales, and to the failure of the runner to give correctly corresponding divisions on the scales. The results may also be in error from inaccuracy in setting and in reading and from parallax when the runner is used. With a good 10-inch rule the result should not be in error by more than $\frac{1}{400}$ to $\frac{1}{500}$,—that is, a 10″ rule is equivalent to a three-place logarithmic table.

28. The Thacher Slide Rule.—The slide rule has been modified in various ways, the object being to secure either increased accuracy in the result or ease in manipulation and in reading. The Thacher* rule gives results that should never be in error by more than one in the fourth figure while the fifth figure can often be found with only a small error. It consists of a wooden base bearing two upright metallic standards with a large circular opening in each, the line joining the centers of the openings being perpendicular to the planes of the standards; these are joined by twenty graduated bars triangular in section, the vertices being outward and the bases forming a cylindrical envelope in which a cylindrical slide moves with either a rotary or a longitudinal motion. The total length of the scales is 60

*Edwin Thacher, C. E. (R. P. I., '63.)

feet while that of the ordinary Mannheim is only 10 inches. The cylindrical slide bears only one scale, the scale corresponding to *C* on the Mannheim being omitted. The principles involved in its use are the same as those already given for the Mannheim.

29. Suggested Modification.—Invert the slide. If a scale similar to *B* inverted was placed on the rule above *A* and another similar to *C* inverted placed below *D*, the ends of all the scales being in the same vertical line,—then we could find with one setting the value of any fractional expression with two factors in the numerator and one in the denominator or with two factors in the denominator and one in the numerator, one of the numbers being variable and one or all of the numbers being squared, the results being read on the rule. The value of the square root of any similar expression could also be found on the rule with one setting. It is believed that a few statements similar to those in Art. 13 could be made to cover all the cases.

With such a rule the product of three numbers, any one or all of them being squared, and also the square root of such a product could be found with one setting. If two similar scales were also placed on the slide, *any* expression involving the multiplication or division of three numbers, any one or all of them being squared and one being variable, and also the square root of such an expression could be determined with one setting, the result being always found on the rule.