This is a reproduction of a library book that was digitized by Google as part of an ongoing effort to preserve the information in books and make it universally accessible.



# ELEMENTARY SLIDE RULE MANUAL 

A Self-Instructing Text Book for Beginners

## BY H. RITOW

PUBLISHED BY
THE FREDERICK POST CO.
3635 NORTH HAMLIN AVENUE CHICAGO, ILL.

$$
\begin{aligned}
& Q A 73 \\
& R 54 \\
& \text { ep. } 2 \\
& \text { Engin. } \\
& \text { Lib. }
\end{aligned}
$$

ENGINEERING LIBRARY
Copyright 1925
The Frederick Post Company Chicago, Ill.


## CONTENTS

Section ..... Page ..... 3

1. Introductory
2. Introductory
3. Description of the Simplest Mannheim Type Slide Rule ..... 5
4. The Logarithmic Scale. Its Meaning ..... 7
5. How to Read a Logarithmic Scale ..... 9
6. How to Set the Indicator or the Slide to a Given Reading on a Logarithmic Scale ..... 17
7. The Decimal Point ..... 19
8. Principles of the Slide Rule ..... 23
9. How to Multiply ..... 27
10. How to Divide ..... 30
11. Problems Involving Multiplication and Division. Ration and Proportion ..... 31
12. Multiplying One Number by Each of a Series ..... 35
13. Dividing One Number by Each of a Series ..... 37
14. Squares and Square Root on the Mannheim Type Slide Rule ..... 40
15. Cubes and Cube Root on the Mannheim Type Slide Rule ..... 44
16. Special Markings on the Logarithmic Scales ..... 46
17. Special Problems ..... 48
18. Accuracy and Limitations of the Mannheim Rule ..... 53
19. Description of the Simplest Ritow Type Slide Rule ..... 55
20. Multiplication and Division on the Ritow Type Slide Rule ..... 57
21. Squares and Square Root on the Ritow Type Slide Rule ..... 59
22. Cubes and Cube Root on the Ritow Type Slide Rule ..... 60
23. Accuracy and Limitations of the Ritow Type Slide Rule ..... 62
24. Outline History of the Development of the Modern Slide Rule ..... 63
25. Adjustment of the Slide Rule ..... 65
26. Definitions ..... 67
27. Selecting a Slide Rule ..... 79
28. Tables and Formulae ..... 81


# ELEMENTARY SLIDE RULE MANUAL 

## Section 1

## INTRODUCTORY

The slide rule up to very recent years (1920), was used almost exclusively by those who had received mathematical training in the subjects of logarithms and trigonometry. Instruction Manuals were, therefore, primarily written for these few, and it is very difficult for those not grounded in mathematics to understand these books.

The author found that most instruction books pay no attention to the subject of how to read the scales of a slide rule, and the few that do, devote little space to this important part of slide rule technique.

Within the last two decades (1905-1924) the use of the slide rule has extended to many fields outside of the purely technical or collegiate. In particular in the United States, many mechanics, merchants and clerks are making good use of the handy calculator, and many others are taking school instruction in its use. High schools are now giving slide rule instruction.

It is for the mechanics, the clerks, the merchants and the high school students that this manual is written. However, another object kept in mind was to prepare a supplementary instruction book to the slide rule manuals now in print, to make it easier for anyone to understand them, even if the reader has little training in mathematics or in the use of scales. Great emphasis is placed on the subject of "How to Read Logarithmic Scales," because a mastery of reading these scales will overcome nine-tenths of all difficulties experienced with slide rules.

An understanding of decimals is all that is necessary for complete comprehension of this manual up to Section 25. In Section 25 will be found illustrations showing the use of sine, tangent, logarithmic and $\log . \log$ scales.

Students should bear in mind, however, that no attempt is made to explain all the ways and methods of using every kind of slide rule. This manual confines itself to the simplest types of Mannheim and Ritow Slide Rules. Since, however, the principles back of these simple rules are the same for all slide rules, the manual will serve as an introduction to any slide rule.

Students should never try to memorize rules, principles or methods. Once these are understood and applied successfully, it will be unnecessary to remember them. Slide rule work becomes second nature after a little practice.

There are parts of the manual that will appear very elementary to some. It should be kept in mind that an effort is being made to present the subject in such a form that it may be grasped by all. Advanced students may skip certain exercises, but it is inadvisable to do so unless the student is very sure of his ground. No one should skip any part of Sections 4, 5, 6 or 7.







## Section 1

## INTRODUCTORY (Continued)

The student should not be surprised if, in solving some of the problems, he finds he cannot obtain the answer as accurately as the given check. The 10 -inch Mannheim Slide Rule gives answers at best with an error of two-tenths of one per cent, and the 10 -inch Ritow Rule is correct only to about one in 1,000 (error of one-tenth per cent). The beginner should be satisfied if he gets his answers correct to three figures. Suppose the check given in the text is 17283 and the answer should read 1725, or 1723 , or 1729 , or even 173 , it is correct.

The student should not be easily discouraged. What appears at first reading to be difficult, may prove at a second or third reading to be extremely simple.

Although the illustrations in the first part of the manual are all of Mannheim Type Rules, the owner of a Ritow Rule will have no difficulty applying the same illustrations to his rule, since the $C$ and $D$ scales on both rules are exactly the same. The student is urged to set his own slide rule to the settings illustrated in the figures. This will greatly help him to understand the text.

The author will be very thankful for any adverse criticisms sent to him, as constructive improvements of the manual may result from such criticism.

The author wishes to express his appreciation of and thanks for the thorough examination of this Manual made by Mr. Walker Hinman, Research Chemist of The Frederick Post Company. Mr. Hinman made many corrections and suggestions which have been followed in all parts of the manual. Mr. Rogers, of Austin High School, Chicago, Ill., a slide rule instructor, was kind enough to point out errors and to make some valuable suggestions.

## Section 2

## DESCRIPTION OF THE MANNHEIM SLIDE RULE

The Mannheim Slide Rule consists of a body, a slide and an indicator (runner or cursor). On the face of the slide rule four scales are marked, two on the slide and two on the body, exactly like those on the slide, the $A, B, C$ and $D$ scales. On the indicator are one or more hairlines. (See Fig. 1.)

The student will notice that the spaces between markings grow smaller from one end to the other. Almost all scales used on slide rules have this quality. He will also notice that between 1 and 2 on the C and D scales, the main divisions are subdivided into ten smaller spaces, between 2 and 4 into five smaller spaces, and between 4 and 10 the main divisions are subdivided only into halves. In the same way the $A$ and $B$ scales from 1 to 2 are şubdivided into 5 ths, from 2 to 5 into halves, and from 5 to 10 into tenths.

These scales are called geometric or logarithmic scales.
(10)

$$
\left.9,1,1,\left.\right|_{E} ^{F}, 1,1,1,1,1,1,\right\}
$$

(11)
(12) $9,1,1,\left.1\right|_{F} ^{F} 1,1,1,1,1,1,1,1$

(14)

## Section 3

## THE LOGARITHMIC SCALE

Now, what are the properties of the Logarithmic Scale?
Let the student measure on Fig. 2, or on his slide rule, the distance from 1 to 2 , from 1 to 3 , from 1 to 4 , from 1 to 5,1 to 6 and so on, and he will find:

|  |  | to 2 about $3^{\prime \prime}$ (on a 10 -inch slide rule) |
| :---: | :---: | :---: |
|  |  | 1 to 3 about $43 / 4$ " ${ }^{\prime \prime}$ |
|  |  | 1 to 4 about 6" |
|  |  | 1 to 5 about $67 / 8{ }^{\prime \prime}$ |
|  |  | 1 to 6 about 73/4" |
|  |  | 1 to 7 about 8 ${ }^{5}$ |
|  |  | 1 to 8 about 9" |
|  |  | 1 to 9 about $91 / 2$ " |
|  |  | 1 to 10 about $97 /{ }^{\prime \prime}$ |

Now, if we add the distance to 2 and that to the No. 3, we get 3 " plus $43 / 4$ " equals $73 / 4^{\prime \prime}$, which is the distance to 6 , the product of 2 by 3 .

In the same way, the distance to 2 added to the distance to 4 gives us three plus six equals $9^{\prime \prime}$, which is the distance to 8 , the product of 2 by 4.

In other words, when the distances from the beginning of a Logarithmic Scale to two or more numbers are added, the distance of the product of the numbers is obtained.

This is the basic quality of the Logarithmic Scale. Thus, Fig. 3 shows how the distance 2 on the $C$ scale, added to the distance 3 on the D scale, equals the distance 6 , the product of 2 by 3.

## EXERCISE 1.

Multiply on your slide rule with the $C$ and $D$ scales.

$$
\begin{aligned}
& 2 \times 3=6 \\
& 2 \times 2=4 \\
& 2 \times 4=8 \\
& 2 \times 5=10 \\
& 2 \times 1.5=3 \\
& 2 \times 2=5 \\
& 2 \times 3.5=7 \\
& 2 \times 4.5=9 \\
& 1.5 \times 2=3 \\
& 1.5 \times 4=6 \\
& 1.5 \times 6=9 \\
& 1.5 \times 3=4.5 \\
& 1.5 \times 5=7.5
\end{aligned}
$$

EXERCISE 2. (For Mannheim Type Slide Rule only)
Multiply on your slide rule with the $A$ and $B$ scales.

| $2 \times 6=12$ | $2 \times 5.5=11$ |
| :--- | :--- |
| $2 \times 7=14$ | $2 \times 6.5=13$ |
| $2 \times 8=16$ | $2 \times 7.5=15$ |
| $2 \times 9=18$ | $2 \times 8.5=17$ |

Sometimes the tenths between two main divisions are numbered from 1 to 9 in smaller figures. These tenths, however, really represent the numbers 11, 12, etc., to 19 on the C and D scales, and 21, 22, 23, etc., to 29 , as well as 11 to 19 on the $R$ scales of the Ritow Rules. The 5th 10 th or half division is sometimes indicated by a dot, sometimes by a small 5.
(15) $\left.\}_{1}^{6} \left\lvert\, \begin{array}{llllllllllllll}F \\ F & & & & & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\end{array}\right.\right\}$


(18) $\left\{\begin{array}{llllllllllllllll}F_{F}^{F} \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ F & 1 & 1\end{array}\right\}$


ELEMENTARY SLIDE RULE MANUAL

## Section 4

## HOW TO READ A LOGARITHMIC SCALE

We have seen that the Logarithmic Scale is divided between major markings, sometimes into tenths, sometimes into fifths, and sometimes into halves.

Sketch 1 illustrates division into tenths.
It is evident that each marking represents a successive tenth between 18 and 19. Thus, the markings from left to right are 180, 181, 182, 183, $184,185,186,187,188,189,190$.

The index or hairline is shown at F-F and the reading is 183 under the hairline.

In Sketch 2, the reading under the hairline is 184.
In Sketch 3, the reading is somewhere between 183 and 184. The eye judges the hairline to be at about six-tenths of the space. The reading is, therefore, 1836.

The $C$ and $D$ scale main divisions between 1 and 2 are usually numbered 1 to 9 in smaller figures. These division marks really represent the numbers $11,12,13,14,15,16,17,18,19$.

EXERCISE 3. (For $10^{\prime \prime}$ Mannheim Type Slide Rules oṇly)
Set the hairline of the indicator successively over the numbers 15 , $16,17,18,19,20,22,24,26,28,30,33,36$ and 40 on the $A$ scale and read each time the number represented on the $D$ scale under the hairline. Record your answers. These should be 1225, 1265, 1303, 1341, $1378,1414,1483,1549,1612,1673,1732,1815,1897,2000$.

EXERCISE 4. (For Ritow Type Slide Rules, with the exception of the Merchants' Rule)
Set the hairline of the runner successively over the numbers 11,12 , $13,14,105,115,125,135,32,33,34,35,36,37,38,39,40,41,42$, 43, 44 of the $R$ scale, and read each time the numbers under the hairline on the $D$ scale. These should be $121,144,169,196,1102,1322,1562$, $1822,1024,1089,1156,1225,1296,1369,1444,1521,1600,1681$, 1764, 1849, 1936.

## EXERCISE 5.

Select the D scale and any other scale of your slide rule and set the indicator somewhere between readings 1 and 2 of the $D$ scale, on an exact numbered division of the other scale, and record the reading on the D scale. Do this for ten or twelve different divisions. Now, set the indicator successively to the recorded readings on the D scale and read the other scale. The readings should be the same exact numbers that you started with.

In the second method of subdivision, the space between two division marks is subdivided into 5 spaces, each one representing two-tenths.

The markings in Sketches 4, 5, 6, 7, 8, 9, therefore, stand for Nos. $200,202,204,206,208$ and 210. The reading in Sketch 4 is 204, in Sketch 5 it is 206, and in Sketch 6 it is halfway between the two, or 205.

# (20) ${ }^{0} 1$ F 

(21) $10111|1| 11$




(25) $3^{3 F} 1$


ELEMENTARYSLIDE RULE MANUAL

In Sketch 7 the reading is somewhere between 204 and 205. The eye judges it to be nearer 205 than 204. Hence, the reading is estimated to be 2047. In Sketch 8 the reading is 2056 (the 6 is estimated). In Sketch 9 the reading is between 200 and 201. It is judged to be 2008.

EXERCISE 6. (For 10" Mannheim Type Slide Rules only)
Set the hairline of the indicator successively over the numbers 5,6 , $7,8,9,10,15$ on the $A$ scale and read, in each case, the number under the hairline on the D scale. Check with 2236, 2449, 2646, 2828, 3000, 3162, 3873.

EXERCISE 7. (For the $10^{\prime \prime}$ Ritow Type Slide Rules, except the Merchants')
Set the hairline of the indicator successively over the numbers 15 , $16,17,18,19,145,155,165,175,185,195,45,50,55$ and 60 on the $R$ scale, and read the numbers represented in each case on the $D$ scale under the hairline. Check with 225, 256, 289, 324, 361, 2102, 2402, 2722, 3062, 3422, 3802, 2025, 2500, 3025, 3600.

EXERCISE 8. (For any slide rule)
Select the D scale and any other scale of your slide rule and set the indicator somewhere between readings 2 and 4 of the $D$ scale, but successively over each number of the other scale, and record the readings on the D scale. Do this for ten or twelve different division marks of the other scale. Now, set the indicator successively over the recorded readings of the D scale and read, in each case, the other scale. You should get the same exact numbers with which you started.

The third method of subdividing a slide rule scale is to subdivide main divisions into two spaces, each representing one-half. Thus, in Sketches 10 to 19, inclusive, each of the ten main divisions between 6 and 7 is subdivided into two parts.

The reading in Sketch 10 is 63, in Sketch 11 it is 64, in Sketch 12 it is 635, and in Sketch 13 the eye judges the reading to be 633.

In Sketch 14 the reading is 60. In Sketch 15 it is 61, in Sketch 16 it is 605 , in Sketch 1.7 it is 604 , in Sketch 18 it is 601, and in Sketch 19 the reading is judged to be 6005, half-way between 600 and 601.

## EXERCISE 9.

If one examines Scale $D$ and notes the subdivisions, one will find that from 1 to 2 the subdivisions are tenths, from 2 to 4 they are each twotenths, and from 4 to 10 they are each five-tenths or halves. (Slide. rules that are not ten inches long will have other subdivisions.)

## EXERCISE 10.

Make a list of all the scales of your slide rule. Examine each scale in succession and make a table giving the method of subdivision for each part of every scale.
$\square$

F
(21) ${ }^{10} 11111|1| 111$

(33) $\int^{10^{F}}|1| 1|1| 1| |^{1}$

(25) $3^{3 B^{\circ}}, 1 \quad 1 \quad 11^{31}$
(26) ${ }^{30} 1$

## ELEMENTARY SLIDE RULE MANUAL

In Sketch 7 the reading is somewhere between 204 and 205. The eye judges it to be nearer 205 than 204. Hence, the reading is estimated to be 2047. In Sketch 8 the reading is 2056 (the 6 is estimated). In Sketch 9 the reading is between 200 and 201. It is judged to be 2008.

EXERCISE 6. (For 10" Mannheim Type Slide Rules only)
Set the hairline of the indicator successively over the numbers 5,6 , $7,8,9,10,15$ on the $A$ scale and read, in each case, the number under the hairline on the D scale. Check with 2236, 2449, 2646, 2828, 3000, 3162, 3873.

EXERCISE 7. (For the 10" Ritow Type Slide Rules, except the Merchants')
Set the hairline of the indicator successively over the numbers 15, $16,17,18,19,145,155,165,175,185,195,45,50,55$ and 60 on the $\mathbf{R}$ scale, and read the numbers represented in each case on the D scale under the hairline. Check with 225, 256, 289, 324, 361, 2102, 2402, $2722,3062,3422,3802,2025,2500,3025,3600$.

## EXERCISE 8. (For any slide rule)

Select the D scale and any other scale of your slide rule and set the indicator somewhere between readings 2 and 4 of the $D$ scale, but successively over each number of the other scale, and record the readings on the D scale. Do this for ten or twelve different division marks of the other scale. Now, set the indicator successively over the recorded readings of the D scale and read, in each case, the other scale. You should get the same exact numbers with which you started.

The third method of subdividing a slide rule scale is to subdivide main divisions into two spaces, each representing one-half. Thus, in Sketches 10 to 19, inclusive, each of the ten main divisions between 6 and 7 is subdivided into two parts.

The reading in Sketch 10 is 63, in Sketch 11 it is 64, in Sketch 12 it is 635, and in Sketch 13 the eye judges the reading to be 633.

In Sketch 14 the reading is 60. In Sketch 15 it is 61, in Sketch 16 it is 605, in Sketch 1.7 it is 604, in Sketch 18 it is 601, and in Sketch 19 the reading is judged to be 6005 , half-way between 600 and 601 .

## EXERCISE 9.

If one examines Scale $D$ and notes the subdivisions, one will find that from 1 to 2 the subdivisions are tenths, from 2 to 4 they are each twotenths, and from 4 to 10 they are each five-tenths or halves. (Slide. rules that are not ten inches long will have other subdivisions.)

## EXERCISE 10.

Make a list of all the scales of your slide rule. Examine each scale in succession and make a table giving the method of subdivision for each part of every scale.
(30) $\left.3\right|_{F} ^{\vec{F}} 1 \quad 1 \quad 1 \quad 1 \quad 15$
(31) $\left.{ }^{3 / 4}, 1,1,1\right]_{\mathrm{F}}^{3}$
(32) $\left.\left.34\right|_{F} ^{F} 11\right|^{3 / 5}$

(34) $\left.34\right|_{F} ^{[F} 1 \quad 1 \quad 1 \quad 1 \quad 37^{5}$
[ 12 ]

ELEMENTARY SLIDE RULE MANUAL

## EXERCISE 11.

Pick a dozen exact numbers on your $D$ scale and set the indicator with the hairline successively over each one of these numbers. Read every other scale under the hairline for each position of the indicator and record readings. Now, reverse the process by picking one of the other scales and setting the indicator successively at readings previously found and recorded. For each of these new positions read all other scales and check up with your previously recorded readings. Repeat the process each time with a new scale until your readings check exactly.

- EXERCISE 12. (For Mannheim Type Rules only)

Set the indicator hairline successively over the following numbers of the $A$ scale and read and record the number under the hairline on the D scale: (left half of A scale first, then the right half of the same scale, with the same numbers) $2,3,5,7,11,13,17,19,23,29,31,37,41,43$, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

The readings on the $D$ scale for the left half of the $A$ scale should be: 1414, 1732, 2236, 2646, 1049, 1140, 1304, 1378, 1517, 1703, 1761, 1924, 2025, 2074, 2168, 2302, 2429, 2470, 2588, 2665, 2702, 2811, 2881, 2983, 3114.

The readings on the $D$ scale for the right half are: 4472, 5477, $707,8367,3317,3606,4123,4359,4796,5385,5568,6033,6403,6557$, 6856, 7280, 7681, 7810, 8185, 8426, 8544, 8888, 91,0, 9434, 9849. Now, set to the answers on the $D$ scale and read the $A$-scale.

- EXERCISE 13. (For Ritow Type Slide Rules only)

Set the indicator hairline successively over the $n$ mbers on the $D$ scale given above, at the beginning of Exercise 12, and record the two readings on the $R$ scale. Check with the answers given in Exercise 12.

Reverse the operation by setting the indicator hairline over your former readings on the $R$ scale and read the $D$ sc le. You should get the numbers you started with.

We will now take up the reading of markings that cause the greatest number of errors, namely, those markings nearest the printed numbers.

Let us examine Sketches 20 to 24. In Sketch 20, the reading is 1000, in 21 it is 11 , in 22 it is 101 , in 23 it is 1001 , and in 24 it is 1005 ; the last figure is estimated.

Let us examine Sketches 25 to 29. In Sketch 25, the reading is 300, in 26 it is 310 , in 27 it is 302 , in 28 it is 301 , in 29 it 3005.

In exactly the same way Sketches 30 to 34 illustrate the readings $34,35,342,341,3405$.

Sketches 35 to 40 illustrate readings 50, 51, 505, 503, 501, 5008. (The last is estimated.)

To help in reading slide rules, the following rule will apply to nontrigonometric scales of the Ritow, and to most of the Mannheim scales:

To read a setting on a scale reading from left to right, mark the number of the main division to the left of the hairline, count the number of tenths between this division and the hairline, and write this number as the next figure of the reading. Now, count the subdivisions as tenths, fifths or halves, depending on the method of subdividing; estimate the space between hairline and subdivision and write the one or two last figures of the reading.

(36) $5,\left.\right|_{1} ^{F}, 1,1,1,1,1,1,1,1,9$

(38) $\left\{\begin{array}{lllllllllllllll}F \\ F & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\end{array} 1,1,1,19\right.$
(39) $\int_{-}^{F} \left\lvert\, \begin{array}{lllllllllllllll}F \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\end{array} 1\right.$


ELEMENTARY SLIDE RULE MANUAL

Thus, in Sketch 41 the main division to the left is 6 , the number of tenths between is 3 , therefore, 63. The estimate of the space is about 4. The reading is, therefore, 634.

In Sketch 42, the main division to the left is 6, the number of tenths is 0 , and the estimate is 3 . The reading is, therefore, 603.

In Sketch 43, the main division to the left is 1 , the number of tenths between is 5 , the number of subdivisions is 2 and the space is estimated at 2. The reading is, therefore, 1522.

In Sketch 44, the main division to the left is 1 , the number of tenths between is 5 , the number of sub-tenths is 0 and the space is estimated at 2. The reading is, therefore, 1502.

If the scale reads from right to left, just substitute right for left in the rule and it will apply. For example, in Sketch 45 the main division to the right is 2 , the number of tenths is 4 , the number of fifths and space between are estimated to be 34 . The reading is, therefore, 2434.

In the same way in Sketch 46, the reading to the right is 2 , the number of tenths is 0 , the fifths and space are estimated to be 13. The reading is 2013.

In Sketch 47 the reading is 2003.
Where divisions are marked with smaller numerals, these merely represent the number of tenths and not the readings.

## EXERCISE 14.

Set the following readings on the $C$ scale successively opposite the right end of the $D$ scale, and read the number on the $D$ scale opposite the left end of the $C$ scale.
$905,915,925,935,945,955,965,975,985,995,475,4775,480$, $4825,485,4875,490,4925,4950,4975,499,320,322,324,326,328$, 330, 332, 334, 336, 240, 242, 244, 246, 248, 250, 195, 196, 197, 198, $199,200,163,164,165,166,167,168,140,141,142,143,144,123$, 124, 125, 126, 109, 110, 111, 112.

The following should be the recorded answers, correct to at least the third place.
$1105,1093,1081,1070,1058,1047,1036,1026,1015,1005,2105$, 2094, 2083, 2073, 2062, 2051, 2041, 2031, 2020, 2010, 2004, 3125, 3106, 3086, 3067, 3049, 3030, 3012, 2994, 2976, 4167, 4132, 4098, 4065, 4032, $4000,5128,5102,5076,5051,5025,5000,6135,6098,6061,6024,5988$, 5952, 7143, 7092, 7042, 6993, 6944, 8130, 8065, 8000, 7937, 9174, 9091, 9009, 8929.

The student should bear in mind that it is perfectly possible for a slide rule worker to read the last figure of any setting, as one of two, or even three numbers. He should not be discouraged if he does not, in the beginning, always get exactly the same reading as this manual leads him to expect. It takes considerable practice for accurate reading.

(42) $\left.1\right|^{F}$





[16]

## Section 5

## HOW TO SET THE RUNNER OR SLIDE TO A GIVEN READING ON A LOGARITHMIC SCALE

As we saw in Section 4, there are three different ways of subdividing Logarithmic Scales. By one method, the smallest divisions represent tenths, by the second method they represent fifths and by the third method they represent halves. We have also seen that all three methods are used in every Logarithmic Scale representing numbers.

These facts the operator should always keep in mind.
To set the indicator to any given reading, one simply applies the principles learned in Section 4.

The rule is as follows:
To set the indicator at a given number on a scale reading from left to right, move the indicator to the right of the division bearing the first 1,2 or 3 figures of the given number, until the hairline has as many. tenths to the right of the division as the next figure of the given number. Now estimate the subdivisions (tenths, fifths or halves) and set the hairline over that part of the scale representing the last one or two figures of the given number. Check the setting by reading it.

## EXERCISE 15.

Set the slide rule indicator to the following readings on the $C$ or $D$ scale:

183, 184, 1836, 204, 206, 205, 2047, 2056, 2008, 63, 64, 635, 633, 60 , $61,605,604,601,6005,1000,11,101,1001,1005,300,310,302,310$, $302,301,3005,34,35,342,341,3405,50,51,505,503,501,5008$, 634, 603, 1522, 1502, 2434, 2013.

Check each reading in succession by the Sketches 1 to 47, inclusive, of Section 4.

To set the slide at a given number, EITHER THE RIGHT OR LEFT END of the scale may be used as index, in place of the hairline in the above rule.

Thus, Sketch 48 shows the left index of the $C$ scale at reading 251 of the D scale, and Sketch 49 shows the right index of the $C$ scale at the same reading.

Sometimes it is more convenient to use one end, sometimes the other. When and why either can be used will be explained in Sections 7, 8, 9 and 10.

In the meantime, the slide rule student should bear in mind that either end of a Logarithmic Scale can be used as index.



## Section 6

## THE DECIMAL POINT

There is one thing the slide rule computer should always remember, and that is, the slide rule does not give the position of the decimal point.

Thus, if we multiply $2.5 \times 3.5$, or $0.25 \times 0.035$, or $2500 \times 35000$ on the slide rule, we get an answer 875 , and we cannot tell from the slide rule whether the answer is 8.75 , or 0.00875 , or 87500000 , or any other number consisting of the three figures 875 with naughts before or after.

The slide rule student should, therefore, always keep in mind:
In slide rule computations, the position of the decimal point in the numbers we multiply with or divide, has no effect on the place of any reading, setting or answer!

There are exceptions to this rule-the figuring of squares and square roots, cubes and cube roots, etc., as well as all figuring with special scales like the $\log \log$ scale, the trigonometric scales and others, all depend on the position of the decimal point. These exceptions make up so small a part of all computations with the slide rule that the above rule should be kept firmly in mind.

If we multiply $5.33 \times 3.21=17.1093$, or $0.0533 \times 0.00321=$ 0.000171093 , the succession of digits or figures in the answers is exactly the same 171093. When we shift a decimal in one or more of the numbers used, the only change made in the answer is to shift the decimal in the answer.

The same can be said of the following:
$\frac{52}{13} \times 7=28$
$\frac{5.2}{0.13} \times 0.07=2.8$
$\frac{0.52}{130} \times 0.7=0.0028$
$\frac{520}{1.3} \times 70=280000$.

The succession of digits or figures is exactly the same in every answer, namely, 28. Sometimes the naughts precede, sometimes they follow the figures. In other words,

In any computation involving multiplication and division only, the position of the decimal point in the initial numbers merely influences the position of the decimal in the answer, but has no effect on the succession of digits or figures.

It is this quality in multiplication and division that we use in slide rule work. One should, therefore, remember that

From the beginning to the end of a slide rule computation, the only matter that counts is the succession of digits in each number.

But how can we tell where to put the decimal point in the answer? That is easily explained.

First of all, in nine out of ten calculations, one knows just where to put the point without thinking. For example, if the weekly wage of an employe is computed and an answer of 4623 obtained, one knows, without thinking, that it must be $\$ 46.23$, and not $\$ 462.30$, nor $\$ 4.62$.

But for the tenth case, the following examples show how to find the decimal point. The rule is first illustrated then stated.

For instance, let us take $2.5 \times 3.5$. We get 875 as answer. You say to yourself, $2 \times 3$ is 6 , therefore, $2.5 \times 3.5$ must be nearer 6 than 60 , or than .6. The decimal point is, therefore, placed at 8.75.

Let us try 0.25 by 0.035 . Again we get 875 . Again we test by first significant digits and we get 6. However, the first number is not 2.5, but 0.25 (one-tenth smaller), and the second is not 3.5, but 0.035 (onehundredth or two decimal points smaller). Hence, the answer must be 1000 th smaller, or three decimal points to the left. One knows, therefore, that the answer must be nearest 0.006 , and it is written 0.00875 .

Now, try $2500 \times 35000$. Again, 875. Again the test gives 6, but this time there are three decimal places more in one fac ${ }^{+}$or and four decimal places more in the other. The answer must have seven decimal places more to the right. It is, therefore, 87500000 .

In division, the extra decimal places of the denominator are subtracted from those of the numerator.

$$
\text { Thus, } \frac{35}{2.5}=14
$$

since $3 \div 2$ is 1.5 , and there is one extra factor of ten in the numerator.

$$
\begin{aligned}
& \frac{0.35}{250}=1.4 \times \frac{1}{10} \times \frac{1}{100}=0.0014 \\
& \frac{3500}{25000}=1.4 \times \quad\left(\frac{1000 \times 1}{10000}\right)=0.14
\end{aligned}
$$

$$
\frac{350}{0.025}=1.4 \times(100 \times 100)=14000
$$

## EXERCISE 16.

The following problems were solved on the slide rule and the answers found are given. Locate the decimal point in each answer.

| $\frac{4.7}{1.3} \times 2.3=832$ | $\frac{9.4}{3.91}=2403$ | $9.4 \times 3.91=3674$ |
| :--- | :--- | :--- |
| $\frac{47 .}{\frac{.13}{.13}=23 .=832}$ | $\frac{0.47 \times 0.023}{1300}=832$ | $\frac{94 .}{0.391}=2403$ |
| $\frac{9400}{3910 .}=2403$ | $\frac{0.0094}{391}=2403$ | $94 \times 0.391=3674$ |


| $y 40 . \times 3910=3674$ |  |
| :--- | :--- |
| 1  <br> $0.78 \times 56 . \times 0.033$  <br> $27 . \times 840$ $=636$ | $\frac{7800 \times 5600 \times 0.033}{0.7 \times 8.4}=636$ |
|  | $=636$ |

In the correct answers, the point will be the following number of figures to the right:

$$
\begin{aligned}
& 1 ; 1 ; 2 ; 4 ;-5(\text { or } 50 \text { 's) } ; 3 ; 1 ;-4 \text { (or } 4 \text { 's to the left) } ; 2 ; 7 ; 1 ;-4 \\
& \text { (or } 40 \text { 's to the left) } 99
\end{aligned}
$$

The negative answer indicates the number of naughts between the decimal point and the first digit.

Notice that dividing the denominator by 100 is the same as adding two decimal places to the answer. The following rule should now be easily understood.

To find the decimal point in a slide rule computation, find, mentally, the approximate answer of performing the same computation with the first significant digits of the numbers; count the extra decimal places to the RIGHT in the NUMERATORS, and to the LEFT in the DENOMINATORS, and subtract therefrom the extra places to the left in the numerators and to the right in the denominators. If the difference is negative, move the decimal point to the left; if positive, move it to the right the number of places the difference indicates.

The following examples will illustrate:

$$
\frac{25 \times 830 \times 0.9 \times 2}{3 \times 75 \times 0.021}=?
$$

The slide rule computation gives 7905. The approximate mental computation gives us:

$$
\frac{2 \times 8 \times 9 \times 2}{3 \times 7 \times 2}=7
$$

25 has 1 extra decimal to the right in the numerator
830 has 2 extra decimals to the right in the numerator
0.021 has 2 extr. decimals to the left in the denominator

5 plus decimal places
0.9 has 1 extra decimal place to the left in the numerator

75 has 1 extra decimal place to the right in the denominator

$$
2 \text { minus decimal places }
$$

The difference $5-2=3$ plus decimal places.
Therefore, the answer is $7.905 \times 1000=7905$.
The following differs from the above only in the decimal points:

$$
\frac{0.025 \times 8.3 \times 90 \times 0.002}{30 \times 0.75 \times 0.00021}=?
$$

The approximate computation is the same and gives 7 again
90 has 1 extra place to the right in the numerator
0.75 has 1 extra place to the left in the denominator 0.0021 has 4 extra places to the left in the denominator

6 plus decimal places
0.025 has 2 extra places to the left in the numerator
0.002 has 3 extra places to the left in the numerator 30 has 1 extra place to the right in the denominator

6 minus decimal places

$$
6-6=0 .
$$

The answer is, therefore, nearest to 7, and it is 7.905.
One more example like the above should suffice:

$$
\frac{2500 \times 0.83 \times 0.09 \times 0.02}{300 \times 7.5 \times 0.21}=
$$

Again, 7 is the approximate answer
2500 has 3 places to the right in the numerator
0.21 has 1 place to the left in the denominator
-4 plus decimal places
0.83 has 1 place to the left in the numerator
0.09 has 2 places to the left in the numerator
0.02 has 2 places to the left in the numerator
300 has 2 places to the right in the denominator
$\frac{7}{7}$ minus places

The difference $4-7=-3$ represents 3 places to the left. The answer is, therefore, $7.905 \div 1000=0.007905$.

The whole operation, explained and illustrated above, for finding the decimal point can be done mentally for the most complicated slide rule computation. Only a little practice is necessary, and finding the decimal points becomes a habit. For the greatest part of all slide rule work, common sense dictates the location of the decimal point without calculation.

If the slide rule worker knows any other method (and there are many) to estimate the approximate answers, such a method would serve equally as well as the above.

Slide rule manuals describe methods of determining the decimal location by watching the number of times the slide is moved out towards the right and left. The author found all these methods with or without counting indicators, much longer, much more confusing, and productive of many errors. Modern slide rule authorities condemn these methods.

The slide rule student is urged not to memorize any rules given in this Manual. If he understands how to apply the rules, and if he performs the necessary exercises, he will never have to think of any rule to use the slide rule.

## Section 7

## PRINCIPLES OF THE SLIDE RULE

We have seen in Section 3 that the outstanding property of Logarithmic Scales is the relation between the number on a marking and the distance of that marking from the beginning of the scale. Adding two such distances gives us a new distance representing the product of the numbers, the logarithmic distances of which were added.

Thus, Figure 4 shows that adding the distance 25 to the distance 3 gives us the distance 75.

In the same figure, opposite the number 2 on the $C$ scale, we have the number 5 on the $D$ scale, or 2.5 times 2. In fact, opposite each reading on the $C$ scale we have a reading on the $D$ scale which is just two and one-half times as large.

We have, therefore, the following two principles of the slide rule:
PRINCIPLE I. Adding logarithmic distances gives the product of the numbers represented.

PRINCIPLE 11. When two identical logarithmic scales are placed next to each other, all the readings on one scale are proportional to the readings directly opposite on the other, the ratios being found opposite the numbers one (the indices). These ratios are reciprocals.

Thus, in Figure 4, the D readings are 2.5 times the C readings, and the $C$ readings are 4 times the D readings. 25 is on the $D$ scale opposite 1 on the $C$ scale, and 4 is on the $C$ scale opposite 1 on the $D$ scale. $4 \times .25=1$. The two ratios, as we see, are reciprocal readings. By reciprocals or reciprocal readings in slide rule work, we mean two numbers whose product is 1 , or 10 , or 100 , or 1000 , or 0.1 , or 0.01 , etc., etc.

Let us examine Figure 4 again, this time from another point of view. First, let us look at the distance to 75 on the D scale. Now, we take away or subtract the distance 3 -we can see it on the $C$ scale-from the distance 75, and what is left? The distance 25 , or the quotient of 75 , divided by 3.

PRINCIPLE III. Subtracting one logarithmic distance from another gives the quotient of the two numbers represented.

Thus, in the same figure on the D scale is the number 8. Subtracting from its distance the distance to 32 on the $C$ scale, and we have again 80
25 left. That is, $\frac{80}{32}=2.5$.
From our first principle we found that adding logarithmic distances multiplies the numbers represented. Therefore, if we add a distance to itself-that is-double it-we are multiplying a number by itself, that is, we are squaring the number.

Thus, in Figure 4 again, we see that the distance 25 added to the distance 25 gives us the distance 625 , which is the square of 25 . Hence, we have

PRINCIPLE IV. To find the square of a number, double its logarithmic distance; to find the cube, triple its logarithmic distance, and 30 on.

One must keep in mind that the logarithmic scale is indefinitely long, but made up of equal lengths, each exactly alike. Now, if we take such an indefinitely long scale and place a single-length scale adjacent to it, anywhere along the line, we will find the beginning and end of this

$24]$
single length are opposite exactly the same readings of the indefinite scale. (See Figure 5.)

If we imagine the single-length scale moved either to the right or to the left, for its whole length, we can see that every part of the singlelength scale will be opposite exactly the same readings on the indefinite scale for either position.

Figure 5 shows this very plainly and that is the reason for
PRINCIPLE V. The left and right ends of a logarithmic scale can be used as indices in place of each other.

We had already stated this principle in No. 5.
We will now place the scale in an inverted or reversed position and let us see what we have:

Figure 6 shows a reversed C scale opposite the D scale with index lines coinciding. Opposite 2 we have 5; opposite 25 we have 4; opposite 3 we have 333 ; opposite 4 we have 25; opposite 5 we have 2; opposite 8 we have 125. In each case the number on the inverted $C$ scale, multiplied by the opposite number of the D scale, gives us 10,100 or 1000 , etc. What does this mean? It means:

PRINCIPLE VI. The numbers on a reversed or inverted scale and on the same scale, not reversed, are reciprocals of each other when the two scales begin and end on the same lines.

Now let us examine Figure 7.
Here we have an inverted C scale opposite the D scale, but not at the same left and right ends. The student can see that the index line of the C scale is opposite 128 on the $D$ scale, and the index line on the D scale is opposite 128 -the same number-on the reversed scale!

The student will also notice that any two numbers under the hairline equal in product the index reading 128. Thus,

$$
\begin{aligned}
& 8 \times 16=128 \\
& 5 \times 25.6=128 \\
& 4 \times 32=128 \\
& 25 \times 5.12=128
\end{aligned}
$$

etc., etc., which really means that we add the reversed scale distance from the number to the right.

We, therefore, have the following two principles which are really a sequel to all the preceding:

PRINCIPLE VII. The distances on a reversed or inverted scale are measured from the right end.

PRINCIPLE VIII. When a reversed and direct logarithmic scale of the same length are placed opposite each other, the reading opposite the indices is the product of any two opposite readings.

The student is warned against memorizing any rules or principles or methods. By faithfully performing all the exercises and trying to understand all the principles, the student will find in a short time that the correct use of the slide rule becomes second nature to him.

The student should notice that reversed scales are all read from right to left.

In Section 6 we explained the following principle:
PRINCIPLE IX. The slide rule does not give the location of the decimal point in any reading or answer, except for log log, trigonometric and special scaleq.



## ELEMENTARY SLIDE RULE MANUAL

## Section 8

HOW TO MULTIPLY .
We have seen in the preceding paragraph that to multiply, add logarithmic distances.

The following figures illustrate a few multiplications.

| In Figure | 8 | $12 \times 6=72$ |
| :--- | ---: | :---: |
| In Figure | 9 | $125 \times 55=6875$ |
| In Figure | 10 | $223 \times 3.33=744$ |

## EXERCISE 17.

Multiply the following on the slide rule and record the answers:

| $13 \times 25$ | $471 \times 1205$ |
| :--- | ---: |
| $18 \times 43$ | $593 \times 1023$ |
| $172 \times 431$ | $2642 \times 253$ |
| $29.02 \times 3.04$ |  |
| $311.5 \times 3.003$ |  |
| $2.715 \times 333$ |  |

The answers should be:

| 325 | 567500 |
| :--- | ---: |
| 774 | 607000 |
| 74100 | 668000 |
| 88.2 |  |
| 936 |  |
| 904 |  |

However, sometimes the answer would appear to come out beyond the right end of the slide rule. It is then we make use of Principle 5. We use the right end of the $C$ scale as index or beginning. The following figures illustrate multiplication when the right index is necessary.
$\begin{array}{lll}\text { In Figure } & 11 & 714 \times 7=5000 \\ \text { In Figure } & 12 & 645 \times .981=633 \\ \text { In Figure } & 13 & 7.675 \times 61.8=475\end{array}$
The student can now see that when we multiply, whether with right or left index, the following is the procedure:

To multiply, set left or right index of the $C$ scale against one factor on the $D$ scale. Move the indicator over the second factor on the $C$ scale and read the answer under the hairline on the $D$ scale.

## EXERCISE 18.

Multiply on the slide rule:

| $91 \times 21$ | $931 \times 115$ |
| :--- | :--- |
| $84 \times 75$ | $8625 \times .291$ |
| $21 \times 56$ | $667 \times 502$ |
| $63 \times 4.75$ | $6.01 \times 905$ |



The answers should be $1911,6300,1176,299,1071,2510,335,5440$.
When multiplying more than two numbers, repeat the operation, using the answer of each product as the first factor of the next product. This answer need not be read nor noted, the indicator hairline holding and märking its position for the nextsoperation.

## EXERCISE 19.

$$
\begin{array}{ll}
5 \times 4 \times 3 & 15 \times 19 \times 24 \\
7 \times 5 \times 6 & 31 \times 1.87 \times 63 \\
8 \times 9 \times 13 & \\
& \\
& 0.921 \times 0.16 \times 843
\end{array}
$$

The answers should be $60,210,936,6840,3650,63100,124.2$.
One must be careful when using indicators that have more than one hairline, to remember which hairline is used for any part of a calculation.

One should not expect a computation with the Mannheim 10-inch Slide Rule to be correct to more than 3 or 4 places, nor expect work with a Ritow Slide Rule to be correct to more than 4 or 5 places.

## Section 9

## HOW TO DIVIDE

The student will remember Principle 3, in Section 7, stated that to divide one number by another, subtract the logarithmic distance of the divisor from that of the dividend.

Figures 8 to 13 , inclusive, illustrate the following divisions:

$$
\begin{array}{ll}
72 \div 6=12 & 5 \div 7=0.714 \\
6875 \div 55=125 & 633 \div 9.81=64.5 \\
744 \div 333=2.23 & 475 \div 61.8=7.675
\end{array}
$$

As can be easily seen, the method for division is the exact counterpart of that for multiplication.

To divide, set the indicator over the dividend on the $D$ scale; move the slide till the divisor on the $C$ scale is under the hairline of the indicator. Read the answer on the $D$ scale opposite the $C$ index.

## EXERCISE 20.

Divide the following:

$$
\begin{gathered}
25 \div 5 \\
73 \div 29 \\
97 \div 84 \\
621 \div 752 \\
462 \div 388 \\
1234 \div 521 \\
2602 \div 401
\end{gathered}
$$

The answers should be: $5,2.518,1.155,0.826,1.191,2.37,6.49$.
Division by more than one number is merely a repetition of the first operation, the indicator being moved over the answer of each operation to act as dividend for the next.

## EXERCISE 21.

Divide by slide rule:

$$
\begin{aligned}
& 2 \div 3 \div 4 \div 5 \\
& 21 \div 32 \div 43 \div 0.55 \\
& 212 \div 323 \div 434 \div 0.555
\end{aligned}
$$

The answers should be:
0.0333
0.02775

$$
0.0273
$$

## EXERCISE 22.

Reduce the following fractions to decimals with the slide rule: •

$$
\begin{aligned}
& 1 / 2,1 / 3,2 / 3,1 / 4,3 / 4 \\
& 1 / 5,2 / 53 / 5,4 / 5 \\
& 1 / 6,5 / 6,1 / 7,2 / 7,3 / 7 \\
& 4 / 7,5 / 7,6 / 7,1 / 8,3 / 8 \\
& 5 / 8,7 / 8,1 / 9,2 / 9,4 / 9,5 / 9 \\
& 7 / 9,8 / 9,1 / 11,2 / 11,3 / 11,4 / 11,5 / 11,6 / 11,7 / 11 \text {, } \\
& 8 / 11,9 / 11,10 / 11
\end{aligned}
$$

Check with the table at the back of the book.

## Section 10

## PROBLEMS INVOLVING MULTIPLICATION AND DIVISION <br> Ratio and Proportion

To multiply and divide in one and the same problem, it is merely necessary to work the problem step by step, the answer from each step being used as the dividend or multiplicand of the next step. It is usually found to be quicker to alternate between multiplications and divisions. In that way, many movements of the slide are avoided.

## EXERCISE 23.

Solve the following by slide rule:

$$
\begin{aligned}
& \frac{5}{3}=? \\
& \frac{5}{3} \times 7=? \\
& \frac{5}{3} \times \frac{7}{9}=? \\
& 5 \times 7=? \\
& 5 \times 7 \div 3=? \\
& (5 \times 7 \div 3) \times 8=? \\
& (5 \times 7 \div 3) \times 8 \div 9=? \\
& \frac{232}{643} \times \frac{591}{481}=? \\
& \frac{992}{418} \times \frac{347}{274} \times \frac{655}{802}=?
\end{aligned}
$$

The answers should be: 1.67, 11.67, 1.296, 35, 11.67, 93.3, 10.37, .443, 2.45.

In all the above problems it is not necessary to record the answers of intermediate operations. The indicator serves the purpose of marking the position of an intermediate answer as long as it is needed.

Long operations should be repeated on Mannheim Slide Rules and should be checked on the folded scales, if computing with a Ritow Slide Rule.

RATIO AND PROPORTION problems reduce themselves usually to the form

$$
\mathbf{X}=\frac{\mathbf{K} \times \mathbf{L}}{\mathbf{M}}=?
$$



To solve this problem, set the indicator over $K$ on the $D$ scale; move the slide till $M$ on the $C$ scale is under the hairline; move the indicator over $L$ on the $C$ scale and read the answer under the hairline on the D scale.

Figure 14 shows the position of slide, indicator and body of rule for the problem.

$$
X=\frac{63 \times 17}{25}=42.8
$$

## EXERCISE 24.

Solve the following proportions and check the answers with those given below:

$$
\begin{aligned}
& \frac{2}{3} \times 9=? \\
& \frac{5}{-} \times 7=? \\
& \frac{8}{8} \times 21 \times ? \\
& \frac{5}{5} \times ? \\
& \frac{15}{64} \times 73=? \\
& \frac{17}{43} \times 132=? \\
& \frac{192}{615} \times 571=?
\end{aligned}
$$

The answers should be $6,5.83,33.6,17.1,52.2,178.3$.
For the next to the last problem it was necessary to mark the position of the intermediate answer with the indicator and move the slide before finishing the problem.

The following exercise gives practice in just that shifting of the slide.

## EXERCISE 25.

Solve the following by slide rule:

$$
\begin{aligned}
& \frac{202}{605} \times 29=? \\
& \frac{503}{792} \times 144=? \\
& \frac{3205}{138} \times 622=? \\
& \frac{55}{3005} \times 784=? \\
& \frac{1111}{930} \times 401=? \\
& \frac{423}{801} \times 1805=? \\
& \frac{604}{2502} \times 5555=?
\end{aligned}
$$

The answers should be $9.68,91.4,14440,143.4,47.9,953,1340$.
The student should always bear in mind Principle $V$ of Section 7. At all times, in computing with the usual slide rule, it is permissible to move the slide the entire length of the scale so that the left index takes the place of the right index or vice versa.

If in all the rules on multiplication and division the rules of the $C$ and D scales were reversed one would still have a correct procedure. However, it is far more efficient to become accustomed to just one way of working with the rule and for this reason all the rules given in Sections 8, 9 and 10 were based on using the D scale as the basic scale upon which the answers are found.

## Section 11

## MULTIPLYING ONE NUMBER BY EACH OF A SERIES

Suppose one wishes to multiply 21 by each of the following numbers: $15,25,35,45,55,65,75,85$ and 95 , one would make use of Principle II of Section 7.

By setting the index of $C$ opposite 21 on the scale $D$ we know that every number on scale $D$ is 21 times the number facing it on Scale $C$.

Figure 15 shows how to obtain the answers.

$$
\begin{array}{ll}
21 \times 15=315 & 21 \times 65=1365 \\
21 \times 25=525 & 21 \times 75=1575 \\
21 \times 35=735 & 21 \times 85=1785 \\
21 \times 45=945 & 21 \times 95=1995 \\
21 \times 55=1155 &
\end{array}
$$

Notice that two positions of the slide were necessary to determine the nine answers, one position with the left index at 21 ; the other with the right index at 21.

## EXERCISE 26.

Find by slide rule the daily payroll for the following men who are paid by the hour at the given rates, and check, by long hand.

| Name | Rate Cents per Hour | No. of Hours |
| :---: | :---: | :---: |
| A. | per | 6 |
| B. | 55 | $61 / 2$ |
| C. | 55 | 7 |
| D. | 55 | $71 / 2$ |
| E. | 55 | 8 |
| F. | 55 | $81 /$ |
| G. | 55 | $81 / 2$ |
| H. | 55 | $83 / 4$ |
| I. | 55 | 9 |
| J. | 55 | $91 / 4$ |
| K. | 55 | $91 / 2$ |
| L. | 43 | 6 |
| M. | 43 | $61 / 2$ |
| N. | 43 | 7 |
| 0. | 43 | $71 / 2$ |
| P. | 43 | 8 |
| Q. | 43 | $81 / 4$ |
| R. | 43 | $81 / 2$ |
| S. | 43 | $83 / 4$ |
| T. | 43 | 9 |
| U. | 43 | $91 / 4$ |
| V. | 43 | $91 / 2$ |
| W. | 37 | 7 |
| X . | 37 | $71 / 2$ |
| Y. | 37 | 8 |
| Z. | 37 | $81 / 2$ |

An important rule in slide rule computations is that fractions must be reduced to decimals before multiplying or dividing, and, in the above exercise, each number must be reduced. For example:

$$
83 / 4=8.75, \text { etc. }
$$

## EXERCISE 27.

Find the piece work payroll of the following by slide rule.

| Name | Article | Quantity | Rate | Amount |
| :---: | :---: | :---: | :---: | :---: |
| A. | X. | 25 | $13 / 4 \mathrm{c}$ |  |
| A. | Y. | 253 | 2/3c |  |
| A. | . Z. | 85 | 41/2c |  |
| B. | $\mathbf{X}$. | 253 | $1 \%$ c |  |
| B. | $\mathbf{Y}$. | 85 | 2/3c |  |
| B. | Z. | 25 | $41 / 2 \mathrm{c}$ |  |
| C. | $\mathbf{X}$. | 85 | $1 \%$ c |  |
| C. | Y. | 25 | 2/3c |  |
| C. | Z. | 253 | 41/2c |  |
| D. | X . | 92 | $18 / 4 \mathrm{c}$ |  |
| D. | Y. | 148 | 2/3c |  |
| D. | Z. | 153 | $41 / 2 \mathrm{c}$ |  |
| E. | X. | 153 | 1\%/4 |  |
| E. | Y. | 92 | 2/3c |  |
| E. | Z. | 148 | $41 / 2 \mathrm{c}$ |  |

Notice that for quick working of this table, one must jump from name to name, first computing all the $X$ amounts, then the $Y$ 's, etc.

Check by long hand.

## Section 12

## DIVIDING ONE NUMBER BY EACH OF A SERIES.

We have seen in Section 7 that, according to Principle VII, if one scale of two identical logarithmic scales is reversed, the reading opposite any index is the product of any two opposite readings on the two scales. See Figure 7.

Or, we can say the number on any one scale equals the index reading divided by the number opposite on the adjacent scale. For example, in Figure 7 the index reading is 128. Opposite 4 we find 32:

$$
\frac{128}{4}=32
$$

also

$$
\frac{128}{32}=4
$$

In the same way we find opposite 2 is 64 , opposite 3 is 42.67, opposite 5 is 25.6 , opposite 6 is 21.33 , etc., and we know that:

$$
\begin{aligned}
& \frac{128}{2}=64 \\
& \frac{128}{3}=42.67 \\
& \frac{128}{4}=32 \\
& \frac{128}{5}=25.6 \\
& \frac{128}{6}=21.33
\end{aligned}
$$

Therefore:
To divide one number by each of a Series, reverse the slide, set the C index opposite the dividend on the $D$ scale and read the quotients on either scale opposite the divisors on the other. Sometimes two settings are necessary, each end of the $\mathbf{C}$ scale being used opposite the dividend on the $D$ scale.

EXERCISE 28.
Divide 1728 by all the numbers from 1 to 30 inclusive.



## ELEMENTARY SLIDE RULE MANUAL

## EXERCISE 29.

How many months will it take to pay off a debt of $\$ 15,000$, if $\$ 125$ and interest is paid every month?

If $\$ 150$ and interest is paid every month?

| ، | 175 | " | " | " | " | " |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| " | 200 | " | " | " | " | " |
| " | 225 | " | " | " | " | ، |
| " | 250 | " | " | " | " | ' |
| " | 275 | " | ، | " | " | " |
| " | 300 | " | " | " | " | " |
| " | 325 | " | " | ، | " | " |
| ، | 350 | " | " | " | " | " |
| " | 375 | " | " | " | " | " |
| " | 400 | " | " | " | " | " |
| " | 425 | " | " | " | ، | " |
| " | 450 | " | " | " | " | " |
| " | 475 | " | " | " | " | " |
| " | 500 | " | " | " | ، | " |
| " | 525 | " | " | " | " | " |
| " | 550 | " | " | " | " | " |
| " | 575 | " | " | " | ، | " |
| " | 600 | " | " | " | ، | " |
| " | 625 | " | " | ، | " | " |
| " | 650 | " | " | " | " |  |
| " | 675 | " | " | " | " | " |
| " | 700 | " | " | " | " | " |
| " | 725 | " | " | " | " | " |
| " | 750 | " | " | " | " | " |
| ، | 775 | " | " | " | " | " |
| " | 800 | " | " | " | ، | " |
| " | 825 | " | " | " | " | " |
| " | 850 | ، | " | ، | ، | *" |
| " | 875 | " | " | " | ' | " |
| " | 900 | ، | " | " | " | " |
| " | 925 | " | " | " | " | " |
| " | 950 | " | " | " | " | ، |
| " | 975 | " | " | " | " | " |
| " | 1000 | " | " | " | " | " |

Check by longhand.

## Section 13

## SQUARES AND SQUARE ROOT ON THE MANNHEIM SLIDE RULE

If the student will examine his rule, he will find that the $A$ and $B$ scales are similar to the $C$ and $D$ scales but the first two are each cut into two halves, of equal length.

By measuring the distances on the A scale from 1 to 2 , from 1 to 3, from 1 to 4 and so on, and doing the same on the $D$ scale, one finds that the D scale distances are exactly twice the A scale distances.

Now suppose we set the Indicator over 15 on the D scale. Then the distance from the left end to the hairline is just twice the distance from the left end of the number 15 on the $A$ scale. Principle IV states that to find the square, double the distance. Hence, the hairline must be over the square of 15 on the A scale. We read it and find it 225, which is the square of 15 . Therefore:

On the Mannheim Slide Rule the $A$ reading, under the hairline, is the square of the $D$ reading, the $B$ reading is the square of the $C$ reading, and therefore the $C$ and $D$ readings are the square roots, respectively, of the $A$ and $B$ readings.

Thus, in Figure 16, 15 on the D scale is opposite 225 on the A scale: $(15)^{2}=225$

In the same way $(4.74)^{2}=22.5$

$$
\begin{aligned}
& (2)^{2}(\text { on the } C \text { scale })=4 \text { (on the B) } \\
& (6.32)^{2}=40 \\
& (3)^{2}=9 \text { (D and A scales) } \\
& (4)^{2}=16 \text { (C and B scales) } \\
& (9.48)^{2}=90 \text { (D and A scales) } \\
& (1.333)^{2}=1.78 \text { (C and B scales) } \\
& (12.65)^{2}=160 \text { (B and C scales). }
\end{aligned}
$$

A second way to find the square of any number, is to multiply the number by itself on the $C$ and $D$ scales. This is a more accurate method.

The slide rule student should remember that every slide rule reading has but one square. But every slide rule reading has two square roots depending on the decimal point.

For example in the figure we can find,

$$
\begin{array}{ll}
\sqrt{9}=3 & \sqrt{90}=9.48 \\
\sqrt{4}=2 & \sqrt{40}=6.32 \\
\sqrt{16}=4 & \sqrt{1.6}=1.26
\end{array}
$$

The two slide rule square roots of the reading 9 are 3 and 9.48; those of the reading 4 are 2 and 6.32 ; those of 16 are 4 and 1.26 .

We have also:

$$
\begin{array}{ll}
\sqrt{900}=30 & \sqrt{9000}=94.8 \\
\sqrt{0.09}=0.3 & \sqrt{0.9}=0.948 \\
\sqrt{400}=20 & \sqrt{4000}=6.32 \\
\sqrt{0.04}=0.2 & \sqrt{.40}=0.632
\end{array}
$$

Now if we think carefully, we see that all numbers from 1 to 9 have square roots from 1 to 3 . All numbers from 9 to 100 have square roots from 3 to 10. All numbers from 100 to 900 have square roots from 10 to 30, and so on. We see, also, that all numbers from 0.09 to 1 , have square roots from 0.3 to 1 , all numbers from 0.09 to 0.01 have square roots from 0.3 to 0.1 , all numbers from 0.0009 to 0.0001 have square roots from 0.03 to 0.01 , etc., etc.

In other words, if the student can remember that the square roots of 9,900 , and 90,000 (even number of naughts) are 3, 30, 300 (half as many naughts); that the square roots of $1,100,10000$ (even number of naughts) are 1, 10, 100 (half as many naughts) ; that the square roots of $0.09,0.0009,0.000009$ (even numbers of decimal places) are 0.3 , $0.03,0.003$; that those of $0.01,0.0001,0.000001$ (even number of decimal places are $0.1,0.01,0.001$; and if the student will remember that all numbers more than one, have square roots that are smaller, and all numbers less than 1 have square roots that are larger, he can make no mistake.

The rule for Mannheim Slide Rules is, therefore:
The square root of numbers between 1 and 10 , multiplied or divided one or more times by 100, are found under the left half of the A scale, on the $D$ scale; the square root of numbers between 10 and 100 , multiplied or divided one or more times by 100, are found under the right half of the $A$ scale on the $D$ scale.

The above examples show that the square root of 9 is 3 , of 900 is 30 , of 90,000 is 300 ; the square root of 0.09 is 0.3 , of 0.0009 is 0.03 , etc.

Again, the square root of 90 is 9.48 , of 9000 it is 94.8 , of 900,000 it is 948 , of 0.9 it is 0.948 , of 0.009 it is 0.0948 , etc.

## EXERCISE 30.

Find the squares of the numbers 1 to 20,101 to 120 , each inclusive. The answers should be $1,4,9,16,25,36,49,64,81,100,121,144,169$, $196,225,256,289,324,361,400$; 10200, 10400, 10610, 10820, 11020, $11240,11450,11660,11880,12100,12320,12540,12770,13000,13220$, 13460, 13690, 13920, 14160, 14400.

## EXERCISE 31.

Find the square roots of 1 to 10 and of $10,20,30,40,50,60,70$, 80, 90, 100.

The answers should be 1, 1.414, 1.732, 2.236, 2.449, 2.646, 2.828, $3.000,3.162,4.472,5.477,6.325,7.071,7.746,8.367,8.944,9.487,10$.

## EXERCISE 32.

Find the square roots of 0.01 to 0.1 inclusive, by hundredths and the square roots of 0.1 to 1 inclusive, by tenths.

Your answers should be respectively each one one-tenth of the corresponding number in Exercise 31.



Figure 17 illustrates a second way of finding the square root with Mannheim Rules. One can see that:

To find the square root of a number, reverse the slide, place the index of the $C$ scale opposite the number, whose root is desired, on the $D$ scale, move the indicator till the same number is found under the hairline on both scales. Use first one then the other end of the $\mathbf{C}$ scale opposite the given number. Each slide rule reading has two possible slide rule square roots.

In the figure for one position, we have $\sqrt{7.5}=2.738$ and in the other position, we have $\sqrt{\prime 75} .=8.65$.

The reason for this method is that the distance between the two indices ( $C$ and D), represents the number on each scale and is cut exactly in two by the reading which is the same on both scales.

## EXERCISE 33.

Find the square roots of 1 to 10 and of $10,20,30,40,50,60,70,80$ 90,100 by the method of the reversed slide.

The fourth power of a number is found by squaring twice; the fourth root is found by taking the square root twice. There are four possible fourth roots of every slide rule reading. The table of roots at the back of the book will help in finding square roots.

## Section 14

CUBES AND CUBE ROOT ON THE MANNHEIM TYPE SLIDE RULE
Figure 18 shows the method of finding the cube of 4.
Either index is set at the given number on the $D$ scale. The answer is found on the $A$ scale, opposite the given number on the $B$ scale.

Now why? Because over the $C$ index must be the square of the number and one is simply multiplying the square by the given number on the $A$ and $B$ scales. The answer must, therefore, be the cube.

## EXERCISE 34.

Find the cubes of all the numbers from 1 to 20 ; record the answers. These should check with the following:

1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, 1331, 1728, 2197, 2744, 3375, 4096, 4913, 5832, 6859, 8000.

Now, to find the cube root the process is reversed and so is the slide, unless the slide rule has a cube root or cube scale.

Figure 19 shows the method of finding the cube root of 64 . If we compare this figure with Figure 18, we will see at once that the principal difference is that the 4 on the $B$ scale has changed places with the index and is now opposite the 4 on the D scale.

By comparing the first half of this figure with Figure 17, we can see that the method is exactly the same as the second square root method, except that the $B$ scale takes the place of the $C$ scale reversed and the answer is read on both B and D scales. The rule is, therefore:

To find the cube root of a number, reverse the slide, set the index of the $D$ scale opposite the given number on the $B$ scale and move the indicator till the hairline covers the same number on $B$ and $D$ scales. There are three slide rule cube roots for every slide rule reading. Usually, first one, then the other $D$ index must be set opposite the number to find all three roots.

Figure 20 shows the method of obtaining $\frac{3}{\sqrt{750}}=9.086$

$$
\begin{aligned}
& \frac{3}{\sqrt{75}}=4.217 \\
& 3 \\
& \sqrt{7.5}=1.957
\end{aligned}
$$

One can easily see that for the first number the distance between the middle $B$ index and the right $D$ index is cut into 3 equal parts by the equal reading, since from $D$ to the reading is twice the distance from $B$ to the reading, which explains why we are getting the cube root. In
the same way the cube root of $\sqrt{75}$ is obtained by cutting the distance from the left $D$ index and the right $B$ index into 3 equal parts again, since $D$ to the reading is twice $B$ to the reading. Also the cube root of 7.5 is obtained by cutting the distance between the $D$ index and the $B$ middle index into 3 equal parts.

The same cube roots for the same positions can be found on the $A$ and $C$ scales, as the figures shows plainly.

Notice that for CUBE root the $D$ index must be placed opposite the number on the $B$ scale.

If we place the $B$ index opposite the number on the $D$ scale we obtain the $2 / 3$ rds power in place of the cube root. (See below.)

## EXERCISE 35.

Find the cube roots of:

$$
\begin{aligned}
& 1 \text { to } 10 \\
& 10 \text { to } 100 \text { by tens. } \\
& 100 \text { to } 1000 \text { by hundreds }
\end{aligned}
$$

The answers should be:
1, 1.260, 1.442, 1.587, 1.710, 1.817, 1.913, 2.000, 2.080, 2.154; 2.154, 2.714, 3.107, 3.420, 3.684, 3.915, 4.121, 4.309, 4.481, 4.642; 4.642, $5.848,6.694,7.368,7.937,8.434,8.879,9.283,9.655,10$.

To find the $3 / 2$ power, find the cube and take the square root. In other words, proceed as for obtaining a cube and read the answer on the D scale, in place of the $\mathbf{A}$ scale. There are two possible square roots to every slide rule reading.

To find the $2 / 3$ power, reverse the slide, set the $B$ index opposite the given number on the $D$ scale and move the indicator till the hairline covers the same number on $B$ and $D$ scales. There are three possible $2 / 3$ roots of every slide rule reading. Usually two $B$ indices must be used to find all three roots.

In Figure 18 we have the cube of $4=64$, and immediately under the 64 we have 8 on the $D$ scale which is 4 raised to the $3 / 2$ power. If we wanted the $3 / 2 \mathrm{~s}$ power of 40 , we would have to try the second position of the slide with the right index at 4 . We would then find under 64, 25.3 on the D scale.

$$
(40)^{3 / 2}=253
$$

In Figure 20 we also have illustrated the method of finding the 2/3rds powers of $8.58,85.8$ and 858 . The answers are found to be:

$$
\begin{aligned}
& (8.58)^{2 / 3}=4.217 \\
& (85.8)^{2 / 3}=19.57 \\
& (858)^{2 / 3}=90.86
\end{aligned}
$$

The student should notice that for $2 / 3$ rds power the $B$ index is set opposite the number on the $D$ scale. He should compare the rules for cube root and for two-thirds powers.

A second method for finding the 2/3rds power is also illustrated in Figure 20. In this method we first find the cube root, then square, the answer being picked out on the A scale after the indicator is set over equal readings on the $B$ and D scales, or it is picked out on the $B$ scale after the indicator is set over equal readings on the $A$ and $C$ scales. Thus, Figure 20 shows that:

$$
\begin{aligned}
& (7.5)^{2 / 3}=3.83 \\
& (75)^{2 / 3}=17.8 \\
& (750)^{2 / 3}=82.6
\end{aligned}
$$

The middle $B$ index in the upper half of Figure 20 is opposite 2.74, and the position shows the following two-thirds powers:

$$
\begin{aligned}
& (2.74)^{2 / 3}=1.957 \\
& (274)^{2 / 3}=42.17
\end{aligned}
$$

The root table in the back of the book will help to pick the proper root for any given number.

## Section 15

## SPECIAL MARKINGS ON THE LOGARITHMIC SCALES

It is frequently a great help on slide rules to have certain readings that are used over and over marked specially. Such markings save time and add accuracy to slide rule work.

The most frequently used special marking is:

$$
\pi=3.1416
$$

$\pi$ is the ratio between the circumference and the diameter of every circle. Its first use is to determine the circumference or diameter of a circle when one of these is given.

Circumference $=\pi$ times the diameter.
Diameter $=$ circumference divided by $\pi$.
The area of a circle equals $\pi$ times the radius squared, or it equals the diameter squared, multiplied by $\pi$ divided by four. It is for this particular calculation which occurs so frequently, that we find on many slide rules the marking:

$$
\pi / 4=0.7854
$$

On Mannheim Rules, the special marking $\pi / 4$ is usually on the $A$ and B scales, on the Ritow Slide Rules it is usually on the C and D scales.

The reason is: On Mannheim Rules the squares are found on the $A$ and $B$ scales; on the Ritow Rules the squares are found on the $C$ and D scales.

Sometimes a slide rule has the marking " or $9 \prime$. This is the fraction of the radius of a circle represented by an arc of the angle of 1 second. This marking has the value of 0.000004848 . It is used in calculations involving trigonometry, especially for small angles. It is also used to find the length of an arc of a circle subtending an angle of so many seconds. This length equals 0.000004848 multiplied by the number of seconds and by the radius of the circle.

Two other similar marks are:

$$
\text { ' or } \eta \text { at } 0.0002909
$$

and

$$
\text { o or } 9 \circ \text { at } 0.0174533
$$

These are used in exactly the same way as the special marking ", except that they are used for angles of so many minutes or so many degrees. The length of a circular arc subtending one minute is 0.0002909 times the radius, and the length of a circular are subtending one degree is 0.0174533 times the radius of the circle.

Other special markings are HP at 0.746 , representing the ratio between horse power and kilowatt.

One horse power equals 746 watts or 0.746 KW .
With this marking one can quickly convert kilowatts into horse powers, or vice versa.

Another useful special marking for engineers is the marking:

$$
2 \mathrm{~g}=64.32
$$

representing twice the acceleration due to the force of gravity.
This figure occurs frequently in engineers' formulae, especially in formulae involving motions or "dynamics." The velocity of water, the question of water heads, the speed of falling bodies, cannon balls, etc., are all computed with the help of this number.

It is a comparatively easy matter to add any special marking to a slide rule desired by its owner. These markings can materially increase the usefulness of a slide rule, if some special purpose requires a number which occurs over and over again in computation.

The author will be glad to advise, if called upon, on the best markings for any particular case.

On some Mannheim Rules, the markings $c$ and $c_{1}$ are found on the C scale. By setting either of these markings opposite the $D$ index (left or right) the $B$ index, or indices, will be found opposite $\pi / 4$ on the $A$ scale. These markings are, therefore, used in computations involving $\pi / 4$.

Thus, to obtain the area of a circle whose diameter is $M$, set the $C$ index opposite $M$ on the $D$ scale and read the area on the $A$ scale opposite $\pi / 4$ on the $B$ scale; or set "c" opposite the D index and read the area on the $A$ scale opposite the $M$ on the $C$ scale. Either operation may be reversed to get the diameter of a circle when given its area.

# ELEMENTARY SLIDE RULE MANUAL 

## Section 16

## SPECIAL PROBLEMS

The following problems illustrate the great variety of uses to whieh the slide rule can be put. Engineering problems are omitted, sinc? most engineers are fully aware of the use of the instrument in their work. The problems are selected in particular consideration of the: limited accuracy of the ordinary Mannheim.
A. Check the following bill for gross errors by slide rule computation:

| 72 | books | 2.25 | $\$ 162.00$ |
| ---: | ---: | ---: | ---: |
| 63 | books | @ | 4.05 |
| 27 | 256.15 |  |  |
| 152 books | books | 0.95 | 25.75 |
|  |  | 0.83 | 126.15 |
|  |  |  | $\$ 570.05$ |

Multiplying by slide rule one finds:
162.00
255.
25.63
126.10

The first and fourth check to the fourth place, but the two middle figures do not check even to the third. Hence these should be rechecked by long hand and the correct figures of $\$ 255.15$ and $\$ 25.65$ are obtained, reducing the bill by $\$ 1.10$ in errors.

Of course, in actual bills one rarely finds an error and the average clerk will not have to re-check more than one in 100 items. He must, however, make a special check of the decimal points in the bill to be sure that some item is neither ten times too much nor one-tenth the proper amount.

It is easily seen how much time can be saved checking bills by slide rule instead of by longhant. The time occupied in doing this work would be reduced fully to one fourth or fifth. A rule that can help check many slide rule problems in this:

The last figure in a longhand product must agree with the product of the last figures of the factors.

Thus, in the above problem: The first item ends with 0 and the product of $2 \times 5$ is 10 ; the second item ends with 5 and the product of $3 \times 5$ is 15 ; the third item ends with 5 and the product of $7 \times 5$ is 35 ; and the last item ends with 6 and the product of $2 \times 3$ is 6 .

This same rule can help find the last figure (4th or 5th) of a slide rule computation. (See Section 17.)

## B. Payroll Computation.

Section 11 explains the method of multiplying one number by a series of numbers. Hence:

To figure a payroll on a time basis, set the index of the $C$ scale opposite the pay per hour on the $D$ scale and read the pay on the $D$ scale opposite the number of hours on the $\mathbf{C}$ scale. Sometimes first the left, then the right index must be used.

This can be calculated accurately enough for the daily payroll on the Mar heim Slide Rule, but for a weekly payroll a $10^{\prime \prime}$ or $20^{\prime \prime}$ Ritow Merch Slide Rule would be necessary to be sure of the last cent in the indis. al weekly amount. However, to check a payroll within 10 cents, even the ordinary Mannheim will do.

To calculate a piecework payroll, the same method can be used as a jue, by substituting the pay per piece for hourly pay, and the number of pieces by the number of hours.

The following rule will help determine the change needed to fill the pay envelopes, where payrolls on an hourly basis are computed to the cent.

For your payroli change, get twice as many pennies, just as many nickels, dimes and $\$ 2$ bills, half as many quarters, half-dollars, single dollars, $\$ 5$ bills and $\$ 10$ bills as there are pay envelopes, and all the rest in $\$ 20$ bills. Fill all envelopes with larger denominations first.

Exercises 26 and 27 illustrate methods of computing the payroll, either on a time or on a piece-work basis. One should, however, bear in mind that the ten-inch Mannheim is really accurate enough only for daily payrolls, i.e., if these are computed to the cent.

For salaried employes on a monthly basis, the procedure is simply that of a problem in proportion. For example, a man receives $\$ 225$ per month and is absent one day. On a month of 24 working days, the 225
employe's pay is reduced $\frac{225}{24}=\$ 9.37$. His pay is, therefore, $\$ 215.63$.

## (See Section 17 on accuracy.)

## C. Computing Business Statistics with the Slide Rule.

If the merchant knows the yearly business done by any department, the cost of materials for that department, the payroll for that department, the overhead in the form of rent, repairs, stationery, tools, supervision, etc., etc., he can, in five minutes, with his slide rule, compute the per cent cost of every part of the outlay and the gross and net profit, all to a greater degree of accuracy than necessary.

| For example: |  |  |
| :---: | :---: | :---: |
| Department A of | \$56,232.65 | business in 1923 |
| Cost of materials | 12,231.87 |  |
| Payroll is. | 14,721.95 |  |
| Rent | 900.00 |  |
| Repairs | 243.52 |  |
| Stationery | 71.83 |  |
| Tools | 51.22 |  |
| Supervision, etc. | 318.75 |  |
| Sales cost. | 5,123.25 |  |
| Net profit | 22,570,26 |  |
|  | \$56,232.65 |  |

(

## ELEMENTARY SLIDE RULEMANUAL

Setting the D index opposite the reading 562 on the $C$ scale, and moving the indicator successively to the following readings on the C scale, we obtain :

| C Scale Readings: | $\%$ |
| :---: | :---: |
| 122 | 21.70 |
| 147 | 26.10 |
| 900 | 1.60 |
| 2,435 | .43 |
| 718 | .13 |
| 512 | .09 |
| 319 | 9.10 |
| 512 | 40.25 |
| 226 |  |
|  | $99.97 \%$ |

The addition shows that there is a total mistake of $.03 \%$, which is small enough for all practical purposes. We simply add the 0.03 to the per cent net profit and we get a net profit of $\$ 40.28 \%$.

## D. Cost and Estimate Computation.

The slide rule is the ideal instrument for this work, as its accuracy is greater than any basis of such figuring could possibly require. Most estimates are inaccurate to more than one per cent, and the cheapest slide rule will give the answers with one per cent accuracy.

For example, let us figure by slide rule the cost of a manufactured article.

A factory found its cost items in per cent as follows:
$\%$
Raw goods . . . . . . . . . 22.0
Payroll ..... . . . . . . . . 26.1
Overhead ............ 2.8
Sales . . . . . . . . . . . . . . . 9.1
Profit . . . . . . . . . . . . . 40.0
$100.0 \%$
Re-figuring, basing all per cent on the cost of raw goods, we get, by dividing by $22 \%$ :

|  | \% |
| :---: | :---: |
| Raw goods. | 0.0 |
| Payroll | 118.6 |
| Overhead | 12.7 |
| Sales | 41.4 |
| Profit | 181.8 |

454.5\%
equals total cost, based on cost of raw goods at $100 \%$.
In other words, by multiplying the cost of raw goods in the particular factory by 4.545 , the price of the finished article is obtained, including a $40 \%$ profit for the manufacturer.

If the raw goods cost $\$ 3.55$, including waste and losses, the price of the finished article should be $3.55 \times 4.545=\$ 16.13$, and the same article could be sold without loss at $40 \%$ less, or $16.13 \times 60 \%=\$ 9.68$.

## E. Per Cent and Interest Computation.

The slide rule is the ideal instrument for all such computations, and the merchant can save himself a great deal of time and clerical help on this one item alone by using the slide rule.

The following are typical per cent, discount and interest problems.
V. How much is $6 \%$ of a given principal?

This is a problem of simple multiplication.
W. What per cent of $\$ 175$ is 33 ?

This is a problem of simple division: $\frac{33}{175}$
$\mathbf{X}$. What is the interest on $\$ 1,525$ for seven months and eleven days, at $6 \%$ a year?

This is a matter of multiplication and division.

$$
1525 \times 6 \% \times(7+11 / 30) \div 12
$$

First, we reduce to decimals with the slide rule, the fraction $\frac{11}{30}=0.367$, then we multiply $1525 \times 6 \% \times \frac{7.367}{12}$, and we get $\$ 56.15$. 30 12
Y. How much is the cost of an article listed at $\$ 12.00$, with a discount of $5 \%$ and $10 \%$ and $20 \%$ ? We know that the final cost is $\$ 12.00 \times 95 \% \times 90 \% \times 80 \%$.
Multiplying the four together, we get $\$ 8.21$ as the cost price.
If the sum is a large sum, it is more accurate to compute the total discount in per cent first, multiply by the list price and subtract the discount in amount from the list price. Thus:

$$
\begin{aligned}
& 95 \% \times 90 \% \times 80 \%=68.4 \%, \text { or }(100-68.4) \%=31.6 \% \text { off. } \\
& 31.6 \% \times \$ 12.00=\$ 3.79 . \\
& \$ 12.00-\$ 3.79=\$ 8.21
\end{aligned}
$$

(See Section 17 on Accuracy.)
Z. Compound interest problems. It is inadvisable for the student to tackle these problems on the ordinary slide rule. An understanding of logarithms is required. Besides, the accuracy of the slide rule is not great enough for these computations. The Ritow Merchants' Log Log Slide Rule computes compound interest as simply as the Mannheim Slide Rule does percentage.

## F. The Slide Rule as a Universal Table of Measures and Weights.

At the back of this Manual are tables of ratios, each one giving the value of one unit of measure in terms of another.

By setting the left or right $D$ index opposite a number on the $C$ scale given by the table, we have on the $C$ and $D$ scales, a table permitting us to find the equivalent of any quantity of either unit in terms of the other. Thus, the table gives 1 cord wood $=128 \mathrm{cu} . \mathrm{ft}$. of stacked wood.

Figure 21 shows the settings to be able to convert any number of cubic feet into cords of wood, or vice versa.

Any reading on the D scale in cords equals the opposite reading on the $C$ scale in cubic feet. Thus, opposite 2.5 on the $D$ scale is 320 on the $C$ scale. Hence:

$$
2.5 \text { cords }=320 \text { feet. }
$$

Of course, in all such cases care should be taken in placing the decimal point.

In exactly the same way, the slide rule can be used to find the equivalent in dollars of any amount of foreign money, or vice versa.

There are many other problems for which the slide rule is a timesaver, but the above should be sufficient to illustrate its wide usefulness.

ELEMENTARY SLIDE RULE MANUAL

## Section 17

## ACCURACY AND LIMITATIONS OF THE MANNHEIM SLIDE RULE

Let us examine the accuracy of the Mannheim Slide Rule 10 inches long. Very careful tests made with a good ten-inch Mannheim, showed that with the greatest care readings on the Mannheim Slide Rule cannot be guaranteed correct to an accuracy greater than that indicated by an error of $1 / 10 \%$ or 1 in 1,000 .

What do we mean by that? Suppose the slide rule answer is 978. $1 / 10 \%$ is 0.9 . Then we know that the answer is, perhaps, 978-0.9 or, perhaps, $978+0.9$. In other words, we know that the answer may be 977, 978 or 979.

Suppose the answer is $542.1 / 10 \%$ is 0.54 . Hence we know that the true answer is somewhere between 541.4 and 542.6 .

Again, suppose the answer is $1262.1 / 10 \%$ is about 1 , and we can feel sure that the true value is between 1261 and 1263.

However, most slide rule calculators do not obtain answers as accurately as just described. The good computer probably gets his answers with an error of less than $2 / 10 \%$, and the average slide rule worker makes errors of less than $1 / 2 \%$ with the ten-inch slide rule.

Any number that is given accurately to four significant figures is accurate to an error of less than $1 / 10 \%$. That is why the ten-inch Mannheim Rule can give you answers to three or four significant figures. For numbers beginning with $4,5,6,7,8$ or 9 , one can only obtain three significant figures accurately. For numbers beginning with 1, 2 or 3, one may get a fourth significant figure with more accuracy for numbers nearer 1, and with less accuracy for numbers nearer 3.

However, this can be kept in mind:
For any given slide rule and computer, the percentual inaccuracy is very nearly a constant. For the ten-inch Mannheim, this percentual inaccuracy varies between $5 / 10 \%$ and $1 / 10 \%$.

For this reason, the use of the Mannheim Slide Rule, 10 inches long, is limited to checking any computation to from $1 / 2 \%$ to $1 / 10 \%$ of the answer, and to all computations requiring no greatè accuracy. For the same reason, the error of a slide rule computation is directly proportional to the answer. The greater the answer, the greater the error.

That is why, in slide rule work, we always try to compute the smaller quantities by slide rule, as for example, in the discount and monthly pay illustrations of the preceding section.

For problems in per cent, in statistical figures, in checking bills, pay rolls, accounts for converting fractions to decimals, or one measure to any other; for all estimates, for finding per cents of profit, cost, sales cost, rent, etc., etc., in any business; for rapid finding of errors in any longhand computation, the ten-inch Mannheim Slide Rule is accurate enough.

The moment, however, a greater accuracy is required than $1 / 10 \%$ or more than four figures are needed in the answer, the ten-inch Mannheim Slide Rule is not sufficiently exact. It is then necessary to use a twenty-inch Mannheim or a ten-inch Ritow Slide Rule, which are twice as accurate as the ten-inch Mannheim rule.

## The accuracy of a slide rule is directly proportional to the graduated

 length of its scales.Thus, the 5 -inch Mannheim is half as accurate as the ten-inch rule; the twenty-inch is twice as accurate, etc. The five-inch rule can be used to an accuracy of from $1 \%$ to $1 / 5 \%$; the twenty-inch rule can be used to an accuracy of $1 / 4 \%$ to $1 / 20 \%$ (or 1 in 2,000 ). The eight-inch rule can be used to an accuracy of from $5 \% \%$ to $1 / 8 \%$, and so on.

The Ritow Students' Rule, because of the double-length scale, has double the accuracy of the same length Mannheim.

The ten-inch Ritow Manifold has the accuracy of a thirty-inch Mannheim, or three times the accuracy of a ten-inch Mannheim, on account of the triple-length scales.

The Ritow Merchants' Rule has ten times the accuracy of a ten-inch Mannheim, since its ten-fold scales are just ten times as long.

A computation involving many movements of the slide (many multiplications and divisions) is less accurate than one with only one or two slide settings.

The accuracy of a slide rule computation depends, therefore, on three factors:

1. The Slide Rule
2. The Computation
3. The Computer

The accuracy of a computation can sometimes be increased by the rule given in Section 16A.

The last figure in a longhand product must agree with the product of the last figures of the factors.

This rule can be used particularly for the determination of the last figure of an exact product, when the slide rule reading will give all the other figures. For example, to multiply 27 by 79, we get, by slide rule, $213+$. We know that the last figure must be 3 , since $7 \times 9$ is 63 . Hence, the answer is 2,133 .

Great care should be taken to use this rule only for the last figure of an exact product; never for an approximate result, and never when the slide rule reading does not give all the other figures of the answer.

## Section 18

## DESCRIPTION OF THE SIMPLEST TYPE RITOW SLIDE RULE

The Ritow Slide Rules differ from the Mannheim Slide Rules in the following details. Wherever, in a Mannheim Type Rule, the graduated length of the rule is cut into two or more equal scales, in the corresponding Ritow Rule the graduated length of scale is multiplied two or more times and folded on the slide rule the same number of times.

Thus, in the simplest or Students' Ritow Rule, we have, in place of the Mannheim half-length $A$ and $B$ scales, the double-length $R$ and $R^{\prime}$ scales. The $C$ and $D$ scales remain the same. (See Fig. 22.)

There are five advantages to the new arrangement.
I. All the usual computations are made on the $C$ and $D$ scales. In the Mannheim Rule, problems involving squares are done principally on the $A$ and $B$ scales, which are half as accurate and with which the computer does not feel quite at home. On the Ritow Rule, computations involving squares are done principally on the $C$ and $D$ scales, giving ease and familiarity of operations and double accuracy.
II. The accuracy of computing with the $R$ and $R^{\prime}$ scales is just the same as that of a twenty-inch Mannheim Rule. Double accuracy again.
III. Squares and square roots are obtained with just twice the accuracy of the same length Mannheim Rule.
IV. The two possible slide rule square roots of any reading are obtained in one setting, whereas two settings are needed with the Mannheim Rule. (See Sections 13 and 20.)
V. Every operation with the folded scales involves an independent check by the C and D scales. (See Section 19.) This gives the slide rule computer certainty of the correctness of his answers.


## Section 19

## MULTIPLICATION AND DIVISION ON THE RITOW TYPE SLIDE RULE

Multiplication and division and combinations of these are done on the Ritow type slide rule in two operations-first, on the C and D scales, exactly as with the Mannheim rule, then with the folded scales ( $R$ and $R^{\prime}$ on the simplest type). The operation with the folded scales is the same as with the $C$ and $D$ scales, except at the reading of the answer. The approximate answer obtained with the $C$ and $D$ scales is used to pick out the nearest answer to it obtained on the folded scales.

The two answers must agree to within $1 / 2 \%$ (on the ten-inch rule).
Figure 23 shows a multiplication of $25 \times 75$, and we find on the R scale, under the hairline, two possible answers- 1,875 and 5,928 . Previous multiplication on the C and D scales gave 187 as the answer, so we know that the correct answer is 1,875 .

The rule is, therefore:
To multiply and divide on a Ritow Slide Rule, first, do so with the C and D scales in the usual way, then repeat on the folded scales with exactly the same procedure, but pick out the answer with the help of the first approximation obtained on the $C$ and $D$ scales.

Figure 23 shows also the multiplications:

$$
\begin{array}{r}
25 \times 23.72=592.8 \\
75 \times 79.08=5,928 \\
79.08 \times 23.72=1,875
\end{array}
$$

The same figure shows the divisions:
$\frac{1875}{75}=25$
$\frac{1875}{23.72}=79.08$
$\frac{5928}{75}=79.08$
$\frac{5928}{23.72}=250$

Repeat Exercises 17 to 29, inclusive, each first on the C and D scales, then on the $R$ and $R^{\prime}$ scales.



## Section 20

## SQUARES AND SQUARE ROOT ON THE RITOW TYPE SLIDE RULE

As we have seen, the $R$ and $R^{\prime}$ scales are just twice as long as the C and D scales and are folded. If the student can imagine a 20 -inch Mannheim cut into two equal lengths and placed one underneath the other, he obtains the Ritow Scales and the reason for the following rules:

To find the square of a number, set the indicator over the number on the $R$ scale and read the $D$ scale.

To find the square root of a number, set the indicator over the number of the $D$ scale and read the $R$ scale. For numbers from 1 to 10 , multiplied or divided by $1,100,100^{2}$, etc., read the root on the upper half ( 1 to 3.162 ); for numbers from 10 to 100, multiplied or divided by $1,100,100^{2}, 100^{3}$, etc., read the lower half of the $R$ scale.

For the reasons for the above rules, the student should read Section 13, also the root table in the back of the Manual.

Thus, in Figure 22, opposite 15, on the $R$ scale, is 225 on the $D$ scale. On the same hairline we find 4.74 on the $R$ scale lower half.

$$
\begin{aligned}
\frac{2}{15} & =225 \\
2 & =22.5
\end{aligned}
$$

Hence, $\sqrt{225}=15 \quad \sqrt{22.5}=4.74$
The student can also check the following in the same figure:

| $2_{2}^{2}$ | $=4$ |
| ---: | :--- |
| $\frac{2}{6.32}$ | $=40$ |
| 2 | $=9$ |
| $\frac{2}{9.48}$ | $=90$, etc., etc. |
| $\sqrt{\overline{4}}$ | $=2$ |
| $\sqrt{40}$ | $=6.32$ |
| $\sqrt{\overline{9}}$ | $=3$ |
| $\sqrt{90}$ | $=9.48$, etc., etc. |

EXERCISE 36. Same as Exercise 30, but with the Ritow Slide Rule.
EXERCISE 37. Same as Exercise 31, but with the Ritow Slide Rule.
EXERCISE 38. Same as Exercise 32, but with the Ritow Slide Rule.
A second method of finding the squares is by multiplying a number by itself on the $R$ and $R^{\prime}$ scales, as explained in Section 19. This is twice as accurate as the first method.

EXERCISE 39. Same as Exercise 30, but by the multiplication method.

## Section 21

## CUBES AND CUBE ROOTS ON THE RITOW TYPE SLIDE RULE

Figure 24 shows the method of finding the cube of the number 4.

$$
4^{3}=64
$$

To find the cube of a number on a Ritow Slide Rule, set the $\mathbf{C}$ index opposite the number on the $R$ scale and read the cube on the $D$ scale, opposite the number on the $C$ scale. To find the $3 / 2$ power, set the C index opposite the number on the $R$ scale, move the indicator over the given number on the $C$ scale and read the answer under the hairline on the $R$ scale. There are two possible $3 / 2$ powers to every slide rule reading.

Figure 24 shows the setting to obtain:
3/2
(4) $=8$
and the setting for $(40)^{3 / 2}=253$.

## EXERCISE 39.

Find the cubes of all the numbers 1 to 20 and check with the answers given in Exercise 34.

## EXERCISE 40.

Find the $3 / 2$ powers of the numbers 1 to 10 and of the numbers 10 , $20,30,40,50,60,70,80,90$ and 100.

The answers should be:
$1,2.828,5.196,8.000,11.18,14.70,18.52,22.63,27.00,31.62$; $31.62,89.44,164.3,253.0,353.6,464.8$, 585.7 , $715.5,853.8,1,000$.

Figure 25 shows the setting for finding the cube root of 64 .
The difference between this setting and that of Figure 24 is that in Figure 25 the slide has been reversed, so that the $C$ index is opposite 64 on the $D$ scale, and the 4 on the $C$ scale is opposite the 4 on the $R$ scale.

The rule for cube root is, therefore:
To find the cube root of a number, reverse the slide and set the $C$ index opposite the number on the $D$ scale, then move the indicator till the hairline covers the same reading on both $C$ and $R$ scales. This reading is one of three possible cube roots. Two settings are necessary, one with each $C$ index opposite the given number on the $D$ scale, to obtain the three possible roots.

In Figure 25, 4 on the reversed scale $C$ is opposite 4 on the $R$ scale.

$$
\sqrt{3}^{64}=4
$$

Also, 1857 on the reversed $C$ scale is opposite 1857 on the $R$ scale. 8

$$
\sqrt{6.4}=1.857
$$

Also, in the second position of the slide reversed, we find 8618 on the reversed C scale opposite 8618 on the $R$ scale.

$$
\frac{8}{\sqrt{640}}=8.618
$$

If the student will observe carefully, he will see that the cube root in each case can be found on the reversed $R^{\prime}$ and $D$ scales as well.

## EXERCISE 41.

Find the cube roots of 1 to 10,10 to 100 by tens, 100 to 1000 by hundreds, and check the answers by those given in Exercise 35.

To find the two-thirds (2/3) power of a number, proceed exactly as in finding the cube root, but take the square of each possible cube root on the $D$ scale.

For example, in Figure 25 over each cube root is the corresponding two-thirds power of $64,6.4$, or 640.

Thus, (64) ${ }^{2 / 3}=16$
and $\quad(640)^{2 / 3}=74.27$

## EXERCISE 42.

Find the two-third powers of the numbers given in Exercise 41. The answers should be: $1,1.587,2.080,2.520,2.924,3.302,3.659,4.000$, 4.327, 4.642; 4.642, 7.368, $9.655,11.70,13.57,15.33,16.98$, 18.57, 20.08 , 21.54; 21.54, 34.20, 44.81, 54.29, 63.00, 71.14, 78.84, 86.18, 93.22, 100.

To help in finding the correct root, the following rules may be kept in mind.

$$
\begin{array}{ll}
\sqrt[3]{ } & 8 \\
\sqrt{1}=1 & \sqrt[3]{8}=2 \\
\sqrt[3]{125}=5 & \sqrt{1000}=10
\end{array}
$$

All numbers between 1 and 8 have cube roots between 1 and 2, and two-thirds powers between 1 and 4. Those between 8 and 125 have cube roots between 2 and 5, and two-thirds powers between 4 and 25. All numbers between 125 and 1000 have cube roots between 5 and 10 , and two-thirds powers between 25 and 100.

Multiplying or dividing a number by 1000 , multiplies or divides the corresponding cube root by 10 and the corresponding two-thirds power by 100.

The root table in the back of the book will prove of help in picking the right root for any problem.

## Section 22

## ACCURACY AND LIMITATIONS OF THE RITOW TYPE SLIDE RULE

As we have seen, the Ritow Type Slide Rule reduces most operations to the $C$ and $D$ scales, simplifies the work involving squares and square roots and other powers and roots, gives at least two-fold accuracy when desired, and gives an independent check in computation in all accurate work.

There is no use to which a Mannheim Type Slide Rule can be put for which a Ritow Type Slide Rule would not serve at least as well. In most cases, the Ritow Type Slide Rule will serve the purpose better.

However, the simple students' type of Ritow Slide Rule has an accuracy only twice as great as that of the Mannheim Type. Its errors average between $1 / 4 \%$ and $1 / 20 \%$ ( 1 in 2000).

For computations requiring greater accuracy, either a Ritow Manifold (threefold accuracy or 1 in 3000 reading accuracy), a Ritow Merchants' Log Log Slide Rule (tenfold accuracy, 1 in 10000 reading accuracy), or a Ritow Cylindrical Slide Rule, which has an accuracy of reading of 1 in 50000 to 1 in 100000 .

The Fuller and the Thacher Slide Rules, though clumsy to use, can be read to an accuracy of 1 in 25000 .

A careful worker with a 20 -inch Ritow Slide Rule can compute any weekly payroll to within two or three cents correctness.

See Section 17 for the uses of the Mannheim Rule, for any one of which the Ritow Rule will be found at least equally serviceable.

See Section 25 for illustrations of some of the rules mentioned.

ELEMENTARY SLIDE RULE MANUAL

## Section 23

## OUTLINE HISTORY OF THE DEVELOPMENT OF THE MODERN SLIDE RULE

The principal authority used for the following outline is the rather complete and documented work by Professor Florian Cajori, of Colorado University, "A History of the Logarithmic Slide Rule," published in 1908 by the Engineering News Publishing Company.

The intimate relation between the slide rule and logarithms is shown clearly by the dates 1614 and 1620. In 1614, John Napier, of Scotland, invented logarithms and six years later, Edmund Gunter, Professor of Astronomy at Gresham College, London, invented what he called "A Logarithmic Line of Numbers." This was nothing less than the basic scale of all slide rules-the logarithmic scale. For many years it was known as "Gunter's Line." The method of using the scale was to measure distances upon it with a pair of dividers and, by adding these or subtracting one from the other, obtaining the product or ratio of two numbers. It was quickly found that the marks left by the dividers on the scale wore the latter and obliterated the markings. Mathematicians tried to overcome this, and we have, in 1630, a reference by Edmund Wingate, in his book of "Natural and Artificial Arithmetic" (London), describing a rule consisting of two Gunter lines sliding one adjacent to the other.

At about the same time, William Oughtred, 1632, published an account of his inventions which included a slide rule of Gunter Lines and a complete account of a circular slide rule with two pointers. This was the first circular slide rule.

For many years mathematicians kept preparing various combinations of logarithmic scales, until the need for being able to read several scales simultaneously led Sir Isaac Newton to think of a "runner," or as we now call it, cursor or indicator. This was about 1675, and it took one hundred years more before John Robertson (England), in 1778, made and explained the use of the "runner."

Slowly the use of the slide rule spread among mathematicians and scientists, until Amédée Mannheim, a French artillery officer, in 1850, invented the well-known combination of half-length and single-length logarithmic scales, and succeeded in having them put on the market by a large firm and adopted by the French artillery. Since that time, Mannheim Slide Rules have been the basis of almost all the newest form of slide rules and have been the principal cause of the widespread use of the device among engineers, architects, scientists and technicians.

Ever since Mannheim's invention, there have been hundreds of improvements patented and tried, in an effort to increase the accuracy of the instrument.

The principle of the folded scale was used with quite some success by Edwin Thacher in his cylindrical slide rule (see Section 25), and the principle of the long scale, spirally wound on a cylinder, was successfully applied by G. Fuller. Both Fuller and Thacher rules, however, proved cumbersome, unwieldly and slow in actual use, and many inventors worked on the problems of using folded scales on the customary form of slide rule. The whole trouble lay in the picking of the answer from among the many possibilities of the number of folds under the hairline.

Very many solutions were found, but none proved practical till H. Ritow patented the combination of single-length and folded scales. The single-length scale gave the index reading with which the more accurate result from the folded scale computation could be picked out and checked. The inventor was fortunate in inducing the Frederick Post Company to make his slide rules, and as a result, these rules are now appearing on the market in the engineers' type, known as the Ritow Manifold and in the merchants' type. The former is a threefold rule, with great convenience in trigonometric and power calculations.

The latter is a tenfold rule with a super log-log scale, enabling root, power and compound interest calculations.

In a short time the extremely accurate Ritow Cylindrical Slide Rule will be put on the market, combining simplicity of operation with onehundredfold accuracy.

At the same time that an effort was made for greater accuracy, new scales were found, such as the log-log scale, which simplified power and root calculations; the inverted scale, which made it unnecessary to reverse the slide and made it possible to multiply three figures at one setting of the slide; the single-fold scale reading from $\pi$ to 10 on to $\pi$ again, saving the computer the resetting of the slide, and many special scales for particular technical needs.

A detailed account of these improvements would take us too far, but we must mention, before closing, the improvements in construction, such as the many devices for keeping the slide rule from warping and from expanding or contracting unevenly; the various successful methods of adjusting the scales to each other, and to the hairline of the "runner," and permitting even movement of the slide rule; the new forms of "runner;" the new forms of slide rules, such as the Duplex and Simplex, both reading on both sides of the rule, the latter with its two parts harking back to the very first slide rule invention. The Simplex form of slide rule construction will shortly be placed on the market by the Frederick Post Company.

There is no doubt whatever that the growing demand for slide rules in the mercantile world will soon give rise to a further advance in slide rule design, both in scale arrangement and in construction.

## Section 24

## ADJUSTMENT OF THE SLIDE RULE

If a slide rule is of good construction, one will find it possible: 1. To adjust the grip of the body on the slide so that the latter will hold any position steadily, yet may be easily moved to any other position.
2. To correct small longitudinal displacements of one part of the body in reference to the other.
3. To fix the hairline in a position giving correct corresponding readings on all scales simultaneously under the hairline.
For the first adjustment, looseh all screws holding one-half the body scales and retighten them till the slide seems to move smoothly, but not loosely in any position. Sometimes scraping or sandpapering the tongues and grooves of the slide is necessary.

For the second adjustment, which is very rarely necessary and which often is not possible-the slide rules in question being made so that all scales are exactly aligned-loosen the holding screws of one-half of the body and, either with a hammer or with your hand, move this part longitudinally till the scales of both parts are exactly aligned.

The third adjustment is made with the screws on the runner, these being losened and the runner or indicator placed so that the hairline covers the index lines of all scales of the body at the same time, either at the right or left end. The indicator is firmly held in this position and the screws are tightened.

In some indicators this adjustment can only be made by bending the spring holding the indicator in place.

Slide rule adjustments should be made very rarely, and most slide rules are used for years without adjusting them.


## Section 25

## DEFINITIONS

The following are the most important terms used in slide rule literature. However, some in particular, those describing the different kinds of scales, are not needed for the simplest types of slide rules. This list of definitions is meant primarily for reference and not for study. The author advises the slide rule student to read the definitions only as far as the definition of "Uniformly Graduated Scale," and to skip the rest till he is ready for a more elaborate slide rule than the simple students' rule. Even if the beginner has bought a better rule, he should first study this manual before attempting to understand the more mathematical, trigonometric, log-log and other scales.

The author cannot emphasize sufficiently that this chapter is to be used primarily for reference.

A slide rule is a calculating instrument with which multiplication and division and kindred arithmetical operations can be quickly performed with the help of scales not uniformly graduated.

There are seven types of slide rule construction now used.

## 1. The Mannheim.

This is the most frequent form of slide rule construction. It consists of a grooved body, a slide and a runner. (See Figure 26.)
2. The Duplex.

This differs from the former in that the body is cut in two parts held together only at the ends and in the use of both faces of the rule for graduation, with, therefore, two transparent faces on the runner. (See Figure 27.)
3. The Circular Slide Rule.

This consists of one or more concentric discs with one or two pointers, the discs and pointers sometimes being turned by a stem winder as are the hands of a watch.
4. The Thacher Slide Rule.

This rule consists of a cylinder and an open framework of bars held by two rings within which the cylinder revolves. (See Figure 29.)
5. The Fuller Slide Rule.

This consists of three cylinders, an inner cylinder with an index rod attached, an axial cylinder with handle and index rod and a graduated cylinder which moves around and up and down the axial cylinder. (See Figure 30.)
6. The Ritow Cylindrical Slide Rule.

This consists of two cylinders mounted on an axis with an index bar or glass, with clamps to fix the cylinders in any position relative to each other or to the index bar or glass. (See Figure 31.)


## 7. The Ritow Simplex Slide Rule.

This consists of two slides and a runner, the slides engaging with each other by tongue and groove and the runner having two transparent faces. Both faces of each slide are graduated just as in the Duplex Rule. One slide corresponds to the body of types 1 and 2. (See Figure 32.)

The body of a slide rule is that part on which the runner slides and by which the runner is held. This applies to types 1, 2 and 7. The body is usually the large part of a slide rule.

The slide is the gliding member of a slide rule, usually a narrow, thin part, with a tongue on each side.

The runner is a sliding member held by the body possessing one or two transparent faces upon which one or more fine index lines are engraved or marked. The runner is sometimes called a cursor and frequently called an indicator. The fine index lines are called hairlines. (See Figure 33.)

The magnifier is a glass cylindrical lens attachable to the runner with the object of magnifying the spaces between division marks to make readings clearer. (See Figure 34.)

The hairline, or indexline, is the fine engraved line on the tfansparent face of the runner, cursor or indicator.
(See pictures of runner types, Figure 33.)
Adjustment screws are used on slide rules principally to keep the groove within which the slide moves at the proper width so that the slide will move freely but not too loosely.

A scale consists of a series of markings or graduations numbered to represent a succession of lengths, quantities or numbers. The marked number of a graduation always bears some direct relation to the distance from the beginning of the scale to the graduation.

When the marked numbers of the graduations are directly proportional to the distance from the beginning of a scale, the scale is uniformly graduated. Thus the inch scale, the centimeter scale, all architects' and engineers' scales are uniformly graduated. The spaces between graduations on uniformly graduated or subdivided scales are exactly equal to each other.

Most of the scales used on slide rules are not uniformly graduated. The spaces between graduations are not equal but gradually grow smaller in the direction from the beginning of the scale taward the end.

For the understanding of the definitions which follow, a knowledge of logarithms is absolutely necessary.

Students are urged to skip this part of Section 25 and to go on to the definitions of "Length of a Logarithmic Scale," unless they are thoroughly grounded in logarithms and trigonometry.

One can learn to use the students' slide rule without any knowledge of mathematics except a little arithmetic and the student need not bother any more about the whys and wherefores of slide rule scales than the automobile driver does about the theories of thermodynamics upon which the design of the engine is based.

The scales used on slide rules are called geometric or logarithmic scales.


In these scales the spaces between successive graduations are inversely proportional to the number represented by the graduation. Thus, one will find the space nearest the number 2 almost exactly half the space nearest the number 1 on every logarithmic scale.

Mathematically, the distances from the number 1 on a logarithmic scale to any graduation is directly proportional to the logarithm - (or rather its mantissa) - of the number that the graduation represents.

The scale of logarithms is not a logarithmic scale. It is a uniformly graduated scale and if the graduated length of this scale is the same as that of a geometric scale, then the readings of the scale of logarithms represent the logarithms of the readings on the geometric scale.

Figure 35 shows Post's Multiphase Slide Rule with the slide, bottom up.

In the figure, the reading 0.4354 is found on the $L$ scale of logarithms directly over the reading 2.725 on the geometric or logarithmic $D$ scale. 0.4354 is the logarithm of 2.725 .

The scale of sines is graduated in such a way that the distance from the left end of the scale to the graduation is proportional to the logarithm of the sine of the angle indicated by the graduation.

Usually the Sine Scale is used in conjunction with the Mannheim A and B Scales.

Thus, in Figure 35 , Sine $25^{\circ}-30^{\prime}$ is found on the A scale to be 0.430 .
In the same way the scale of tangents is graduated in proportion to the logarithms of the tangents of the angles indicated by the graduations. The scale of tangents is frequently used in conjunction with the C and D scales. Thus, in Figure 35, tangent $25^{\circ}-30^{\prime}$ is seen on the D scale to be 0.477 .

Sometimes the sine scale is shown in two halves, the first half reading from about $34^{\prime}$ to nearly $6^{\circ}$. As for these small angles, the sine and tangent are almost equal, this scale can also be used for the tangents of small angles.

Thus in Figure 36, which shows a Post's Ritow Manifold Slide Rule, each half of the folded sine scale is used with the D scale. Sine $25^{\circ}-30^{\prime}$ is read to be 0.4305 on the $D$ scale and the sine of $2^{\circ}-28^{\prime}$ is found on the D scale at the same point as 0.04305 . The tangent of $2^{\circ}-28^{\prime}$ is the same. For the first half of the folded sine scale there is always a naught after the decimal followed by the slide rule reading. For the second half of the sine scale the slide rule reading comes right after the decimal point.

An inverted scale is sometimes called a reversed scale and somtimes a scale of reciprocals. (See Figure 37.)

By placing an inverted scale directly over the corresponding logarithmic scale, readings on both scales under the hairline will be exact reciprocals. By reciprocals, or reciprocal readings, in slide rule work, we mean two numbers whose product is 1 or 10 or 100 or 1000 , or 0.1 or 0.01 , or etc., etc., that is, any two numbers whose product is 1 followed or preceded by naughts are slide rule reciprocals.

Figure 38 is the upper face of a Post's Multiphase Slide Rule.
In the illustration, the reading 325 on the $C$ scale is directly under the reciprocal 3.078 under the hairline on the $C$ I scale. In the same way 2 is over 5,25 is over 4, 3.333 is over 3, etc., etc.



FIG. 40

By taking the slide and reversing it in the groove, one obtains a series of reversed scales.

The student should compare scale C, of Figure 39, with scale CI, of Figure 38. He will notice that the only difference is that the numbers in the C scale are upside down. Otherwise the two scales are identical.

The length of a logarithmic scale is theoretically infinite. (See Figure 40.)

However, examination of such an infinite scale shows that the infinite scale consists of a series of scales all of exactly the same length and of the same graduations. It is this length that we call the length of a logarithmic scale. The length of a logarithmic scale is, therefore, the distance between the readings 1 and 10 , or 10 and 100 , or between 100 and 1000 .

In 10 -inch slide rules the length of the principal logarithmic scales of the rule are 25 centimeters, in 20 -inch slide rules the length is 50 centimeters, in 5 -inch slide rules the length is $121 / 2$ centimeters, etc.

This length in each case is called the graduated length of the rule. The graduated length is therefore always just a little less than the total length of the slide rule.

Sometimes the scales are extended a little beyond the left and right ends of the graduated length. See the pictures of folded scales. (See Figures 36, 46, 47.) The student should therefore always keep carefully in mind that the graduated length of a slide rule is the length of the C scale or D scale of the Mannheim rule, usually $121 / 2$ centimeters in 5 -inch slide rules, 20 centimeters in 8 -inch slide rules, 25 centimeters in 10 -inch slide rules and 50 centimeters in 20 -inch slide rules.

The left index line of a slide rule is the left end of the graduated length of the rule.

The right index line of a slide rule is the right end of the graduated length of the rule.

The student should note that on many slide rules there are graduations extending outside the index lines and he should not confuse the index lines with the last graduations.

It is frequently useful to combine logarithmic scales of various lengths on the same slide rule. To help understand such combinations, the following definitions will be useful.

A single length logarithmic scale is a scale of the graduated length of the rule. (See Figure 41.)

A half length logarithmic scale is a scale half the graduated length of the rule. (See Figure 42.)

A one-third length logarithmic scale is a scale one-third the graduated length of the slide rule. (See Figure 43.)

Notice that two half length scales and three one-third length scales are used in the graduated length of the rule shown in Figure 38.

A folded scale is a logarithmic scale that begins or ends, or begins and ends at a reading which is not one (1) nor ten (10) nor powers of ten.

A one-fold or single-fold scale is a single length logarithmic scale beginning and ending at some arbitrary marking of the logarithmic scale.

In Figure 44 the single fold scale begins and ends in 5.


See also the picture of the infinite logarithmic scale (Figure 40), which shows how a single fold scale can be cut out of an infinite scale by simply cutting at readings which are neither 1 nor 10.

Single-fold scales very frequently begin and end at $\pi$. (See Figure 45.)

A double length logarithmic scale is of twice the graduated length of the rule. Hence, in order to show it on the slide rule, it must be cut into two folds which are placed one below the other. It is therefore also called a two-fold scale. (See Figure 46.)

In the same way a triple length, or 3 -fold scale, is one of three times the graduated length of the slide rule cut into 3 folds, each equal to the graduated length of the rule. (See Figure 47.)

The student will observe that the folds are frequently extended outside the graduated length to make the reading of the scales easier. The scale at the right end of one fold is duplicated in the beginning at the left of the next fold.

By placing a two-fold, three-fold and single-length scale, one underneath the other with index lines coinciding, we find the numbers on the single-length scale are the squares of the numbers on the two-fold scale and the cubes of the numbers on the three-fold scale. (See Figure 36.)

Thus the numbers under the hairline in Figure 36 are:
$1.2 \quad 2.585 \quad 5.570 \quad$ on the A scale.

On the $R$ scale we have 1.315 and 4.155 , and 1.728 on the $D$ scale.

$$
\text { and } \begin{array}{rlr}
1.2^{3} & =1.728 & 2.585^{3}=17.28 \\
5.57^{3}=172.8 \\
1.315^{2} & =1.728 \\
4.155^{2} & =17.28
\end{array}
$$

Figure 48 shows a ten-fold scale, as used on Post's Ritow Merchants' Log Log Slide Rule.

A helical scale is a scale wound helically around a cylinder.
We will now describe the $\log \log$ scale. The student is warned that a part of this description requires a thorough understanding of logarithms and it is very inadvisable for anyone not grounded in mathematics to attempt to understand the explanation. The student should also keep in mind that he can learn to use the log log scale with perfect familiarity without any understanding whatever of the mathematical basis of the scale.

There are two main divisions of the $\log \log$ scale. We will call one the $\mathrm{E}_{\mathrm{x}}+$ scale and the other the $\mathrm{E}^{-x}$. scale. (See Figure 49.)

The $E+x$ scale is a scale in which the graduations are placed at a distance from the index line exactly proportional to the logarithms to the base 10 of the logarithm to the base (Napierian base) $\mathrm{e}=2.7183$ of the reading. That is, if $y=$ the reading of the graduation, the distance from the left equals $\log { }_{10}(\log y)$. Figure 49 shows the $\log \log$ scales used on Post's Ritow Merchants' Log Log Slide Rule.

As $\log \log$ scales are needed to a length equal to 2 or more times the graduated length of the rule, the $\log \log$ scales are almost always folded.

Most $\log \log$ slide rules show 2 or 3 folds of the $E+x$ scale. However, the $\mathrm{E}^{-1}$ scale is just as useful as the other and supplements the other particularly in compound interest computations.


The $\mathrm{E}^{-\mathrm{x}}$ scales are graduated so that the distance from the index line is directly proportional to the logarithm to the base 10 of the minus logarithm to the base $e$ of the reading. That is, if $y=$ the reading of the graduation, its distance from the left index $=\log _{10}(-\log \mathrm{e} y)$.

Log log scales have three conspicuous qualities.
First. Any reading on a $\log \log$ scale represents $e^{+x}$ or $e^{-x}$ of the corresponding reading $X$ on the single length logarithmic scale. Thus, in the figure, the numbers under the hairline just over the reading 2 on the single length scale are:

$$
\begin{aligned}
1.00202 & =\mathrm{e}^{0.002} \\
1.0202 & =\mathrm{e}^{0.02} \\
1.2214 & =\mathrm{e}^{0.2} \\
\text { and } 7.3891 & =\mathrm{e}^{2}
\end{aligned}
$$

and the readings on the $E-x$ scale are

$$
\begin{aligned}
& 0.998002=e^{-0.002} \\
& 0.980200=e^{-0.02} \\
& 0.81873=e^{-0.02} \\
& 0.13534=e^{-2}
\end{aligned}
$$

The second quality is: the numbers on the $\mathrm{E}^{+x}$ scale and the corresponding number on the $\mathrm{E}^{-x}$ scale are exact reciprocals, one of the other. Thus, the two series of numbers given above are exact reciprocals.

The third and most important quality is that of the finding of powers and roots.

By placing the left or right index of the single length logarithmic scale against any number on either $\mathrm{E}^{+\times}$or $\mathrm{E}^{-x}$ scale, we will find the power of that number on the $\mathrm{E}^{+x}$ or $\mathrm{E}^{-x}$ scale directly opposite the index of the power on the single length scale.

Thus, in figure 49 the right index line of the $D$ scale is placed opposite the number 2 of the $\mathrm{E}^{+x}$ scale, and one can read over the number 2 of the single length scale the square of $2=4$ over 3 , the cube $=8$ over 4 , the $4^{\text {th }}$ power $=16$ over 5 , the $5^{\text {th }}$ power $=32$, etc., etc. At the same time on the $E^{-x}$ scale, one can read the powers of 0.5 .

| On the single |
| :---: |
| length scale |
| over |
| " |
| " |
| 3 |
| " |
| " |
| 4 |
| " |

$$
\begin{aligned}
& \text { On the } E^{-x} \text { Scale } \\
& 0.5^{2}=0.25 \\
& 0.5^{3}=0.125 \\
& 0.5^{4}=0.625 \\
& 0.5^{5}=0.03125 \\
& 0.5^{6}=0.015625
\end{aligned}
$$

etc., etc.
In exactly the same way the roots of any number can be found on the $\mathrm{E}^{+x}$ and $\mathrm{E}^{-x}$ scales over fractional numbers of the single length logarithmic scale with the index of the latter under the number whose root or fractional power is to be obtained.

Thus, in the figure over 5 on the single length scale we find $\sqrt{2}=1.414$, over .3333 we find $\sqrt[3]{2}=1.26$, over 0.25 on the single length scale we find $4 \sqrt{2}=1.189$, etc., etc.

Notice that $2^{0.5}=21 / 2=1.414$ is directly over

$$
2^{5}=32 \text { and that }
$$

$2^{0.05}=20 \sqrt{2}=1.0353$ is directly over $2^{1 / 2}$ and that $2^{0.005}=200 \sqrt{2}=1.00347$ is directly over ${ }^{20} \sqrt{2}$.
In other words, the four readings under the hairline on the $\mathrm{E}+\mathrm{x}$ scale represent successive tenth powers or roots of the same number. For any power of a number we must look on a fold below the number, or to its right on the same fold. For any root of a number, we must look on a fold above the number, or to its left on the same fold. The same is true in the $\mathrm{E}^{-\mathrm{x}}$ scale, as the following readings over the number 2 on the D scale show: $0.5^{0.005}=0.99654,0.5^{0.05}=0.9659,0.5^{0.5}=$ $\sqrt{0.5}=0.7071$ and $0.5^{5}=0.03125$

The easiest way to work with the $\log \log$ scales $\mathrm{E}+\mathrm{x}$ and $\mathrm{E}-\mathrm{x}$ in finding a power or a root, is to reduce all fractional powers and all roots to decimal indices, setting the left or right end of the D scale opposite the number whose power or root is desired, and to read the answer on the proper fold opposite the decimal power or root reading of the D scale.

$$
\begin{aligned}
& \text { Square root }=0.5 \text { power index } \\
& \text { Cube root }=0.3333 \text { power index } \\
& \text { Fourth root }=0.25 \\
& \text { Three halves } \\
& \quad \text { power }=1.5
\end{aligned}
$$

All $\log \log$ readings include the decimal point!

## Section 26

## SELECTING A SLIDE RULE

There are three factors the slide rule student should consider in . buying his rule.

First. For what purpose does he expect to use the slide rule?
Second. What accuracy does he need for his computations?
Third. What construction, adjustment and shrinkage features should the student look for in the slide rule he is buying?
If a cheap rule is desired for preliminary study, a Students' Mannheim or Ritow Slide Rule as made by the Frederick Post Company will do. These do not cost much and will serve well enough to make one acquainted with the basic principles and methods of the slide rule.

If the student is preparing an engineering career, he should buy a slide rule that contains trigonometric scales and if possible, also power and root scales. Most Mannheim Rules are supplied with trigonometric scales on the back of the slide. However, the following rules have some form of reversed scale and some means of obtaining cubes and cube roots.

> Post's Ritow Manifold Slide Rule.
> Post's Multiphase Slide Rule.
> Post's Multiphase Duplex Slide Rule.

ELEMENTARY SLIDE RULE MANUAL

Of the above by far the most accurate and convenient is the Ritow Manifold which combines the accuracy of a thirty-inch Mannheim with special trigonometric power and root advantages and which permits almost all operations to be performed with the same three C, D and I scales.

The Ritow Manifold gives any trigonometric function of an angle in one setting and permits obtaining all the corresponding cubes and cube roots, squares and square roots, three halves and two-thirds powers all in one setting. Fifth powers and roots, sixth powers and roots are obtained in one setting.

The Manifold is, however, wider than the other rules and a little more expensive.

The Multiphase and Duplex Rules share with the Manifold the quality of enabling multiplications with three factors in one setting with the help of an inverted scale. Though not as accurate nor as convenient as the Manifold, these rules are not as wide and are somewhat cheaper.

The Duplex Rules have single fold scales which make it unnecessary to shift the slide from one index to the other and are, therefore, the most convenient rules for multiplication and division with many factors.

The technical student, engineer or architect should buy one of the slide rules mentioned above.

The merchant who must have more accuracy than that given by the ordinary Mannheim, or who wishes to compute interest, annuities, amortization rates, insurance rates, etc., etc., should buy Post's Ritow Merchants' Log Log Slide Rule which has the accuracy of a hundred-inch slide rule combined with an eight-fold $\log \log$ scale that makes the computation of compound interest so simple that any clerk can easily and quickly find the desired result.

The same rule is of particular advantage to mathematicians and engineers who have need of computing powers and roots frequently or who make use of hyperbolic functions or of the quantities $e^{+x}$ or $e^{-x}$, for the rule gives any root in one setting or permits finding $e^{+x}$ or $e^{-x}$ in any desired combination.

There are special slide rules for special purposes, such as the Electrical and Photoengravers' Slide Rules made by the Frederick Post Company.

Now in regard to the accuracy of the slide rule, the buyer should keep in mind that the least accurate are the circular slide rules and the five-inch Mannheim Rules. The ten-inch Mannheim type, including Multiphase and Duplex Rules, are twice as accurate. The ten-inch Students' Ritow Slide Rule and the twenty-inch Mannheim Type Rules are four times as accurate. The ten-inch Ritow Manifold is six times as accurate, the twenty-inch Ritow Manifold is twelve times as accurate, the ten-inch Ritow Merchants' Log Log Rule is twenty times as accurate and the twenty-inch Merchants' Log Log Slide Rule is forty times as accurate as the five-inch Mannheim.

The average error with the five-inch Mannheim is less than $1 \%$; the average error with the twenty-inch Ritow Log Log Slide Rule is less than $0.025 \%$, or less than 1 in 4000 and can be reduced almost as little as 1 in 20,000.

The most accurate rule of all is the Ritow Cylindrical Slide Rule, which has an error of less than 1 in 40000.

In reference to construction, the slide rule buyer should be careful to pick out among the many slide rules of one type, the particular rule which has the best fit of the left and right ends of the graduations when the slide is set at 1. He should also be careful to pick an indicator on which the hairline accurately covers all indices at one time at either end of the slide rule. The slide should be completely removed for a careful examination of the rule. A little time in examining the slide rule will quickly reveal any looseness of construction or warping or shrinking of any part of the body or slide.

If the buyer's eyes are weak, it is best to buy a magnifying indicator with the rule.

## Section 27 <br> TABLES AND FORMULAE WEIGHTS AND MEASURES

1 Square Mile $=640$ acres.
1 Acre $=43,560$ square feet.
1 Rod $=51 / 2$ yards $=1$ pole $=1$ perch.
1 Square Rod $=301 / 4$ square yards.
1 Chain $=4$ rods $=100$ links.
1 Square Chain = 16 square rods.
1 Acre $=10$ square chains.
1 Knot $=1.152$ miles per hour $=1$ Nautical mile per hour.
1 Short Ton $=2,000$ pounds.
1 Long Ton $=2,240$ pounds.
1 Cord Wood $=128$ cubic feet stacked wood.
12 Feet Board Measure = 1 cubic foot.
1 Span = 9 inches.
1 Hand $=4$ inches.
1 Furlong $=40$ rods.
1 League $=3$ miles.
1 Fathom $=6$ feet.
1 "Section" = 1 square mile.
1 Circular Inch $=0.7854$ square inches.
$=$ area of circle 1 inch diameter.
1 Circular Mil = area of circle .001 inch in diameter.
$=0.000001$ circular inch.
1 Vara (Texas) $=331 / 3$ inches.
1 pound avoirdupois $=16$ ounces.

$$
=7,000 \text { grains }
$$

$=256$ drams.
1 Stone $=14$ pounds.
1 Hundredweight (cwt.) = 112 pounds.
1 Troy Pound = 12 ounces (Troy).
(also apothecary) (also apothecary)

$$
\begin{array}{ll}
5760 \text { grains. } & \text { The grain is the same } \\
=0.8229 \text { pounds. } & \text { in all systems. }
\end{array}
$$

avoirdupois
$\left.\begin{array}{l}1 \text { scruple }=20 \text { grains } \\ 1 \text { dram }=60 \text { grains }\end{array}\right\}$ apothecary
1 Carat $=200$ milligrams $=0.007054$ ounces (avoirdupois).
1 Cubic Foot $=7.4805$ gallons.
1 Gallon (U. S.) $=4$ quarts.
$=8$ pints.
$=32$ gills.
$=231$ cubic inches.

$$
\begin{aligned}
& 1 \text { Imperial Gallon }(\text { British })=277.418 \text { cu. in. } \\
& 1 \text { Bushel (U. S.) }=4 \text { pecks. } \\
&=32 \text { quarts. } \\
&=64 \text { pints. } \\
&=2150.42 \text { cubic inches. } \\
& 1 \begin{aligned}
& \text { Imperial Bushel }=2219.344 \text { cubic inches. } \\
& \text { (British) }
\end{aligned}
\end{aligned}
$$

## ENGINEERING UNITS

1 Kilogram per square centimeter $=14.223$ pounds per square inch.
$\mathbf{g}=$ acceleration due to gravity. $=32.174$ feet per sec. per sec.
$=9.806$ meters per sec. per. sec.
1 Atmosphere pressure $=14.7$ pounds per square inch.
$=33.9$ feet water pressure.
$=29.92$ inches mercury pressure.
$=760$ millimeters mercury pressure.
1 Watt $=1$ Joule per second.
$=1$ ampere volt $=0.7376$ foot pounds per second.
1 British Horsepower $=745.7$ watts $=550$ foot pounds per second.
1 Metric Horsepower $=73.5 .5$ watts.
$=7500$ kilograms centimeters per second.
1 Gram Calorie (mean) $=3.086$ foot pounds.

$$
=4.183 \text { Joules. }
$$

$$
=0.003968 \text { (mean) British Thermal Units. }
$$

(B. T. U.)

1 British Thermal Unit (B. T. U.) $=777.52$ foot pounds.
1 Boiler Horsepower = equivalent evaporation of 34.5 pounds of water from and at 212 degrees per hour.

$$
=33,479 \text { B. T. U. (British Thermal Units). }
$$

1 Kilogram-Centimeter $=0.07233$ foot pounds.
1 Joule $=0.7376$ foot pounds.
$=10^{7} \mathrm{ergs}=10^{7}$ dyne cm.
1 Miner's Inch $=1 / 40 \mathrm{cu} . \mathrm{ft}$. to $1 / 50 \mathrm{cu} . \mathrm{ft}$. water per second. Temperature of freezing water $=0^{\circ}$ centigrade.

$$
=32^{\circ} \text { Fahrenheit. }
$$

$=0^{\circ}$ Réaumur
Temperature of boiling water $=100^{\circ}$ centigrade.
$=212^{\circ}$ Fahrenheit.
$=80^{\circ}$ Réaumur.
Absolute 0 of no heat $=-273^{\circ}$ centigrade.
$=-459.4^{\circ}$ Fahrenheit.
$=-218.4^{\circ}$ Réaumur.
$1^{\circ}$ Centigrade $=9 / 5^{\circ}$ Fahrenheit $=4 / 5^{\circ}$ Réaumur.

$$
\begin{array}{ll}
\mathrm{C}=5 / 9^{\circ} \times(\mathrm{F}-32) & \mathrm{F}=9 / 5 \mathrm{C}+32 \\
\mathrm{R}=4 / 9^{\circ} \times(\mathrm{F}-32) & \mathrm{F}=9 / 4 \mathrm{R}+32
\end{array}
$$

Specific Gravity $=\frac{\text { Weight of body }}{\text { Weight of equal }}$

$$
=\frac{\text { Weight of } 1 \text { cubic foot }}{62.5 \mathrm{lbs} .}
$$

$\frac{5 \mathrm{~T}^{\circ}+1000}{$|  Specific Gravity of liquid  |
| :---: |
| 1000 |$\frac{170}{170 \pm \mathrm{B}^{\circ}}=\frac{400}{400 \pm \mathrm{X}^{\circ}}=\frac{140}{130+\mathrm{L}^{\circ}}=\frac{145}{145-\mathrm{H}^{\circ}}=}$

$=$ Specific gravity of liquid where:
$\mathrm{T}^{\circ}=$ Degrees Twaddell
$\mathrm{B}^{\circ}=$ " Beck
$\mathbf{X}^{\circ}=\quad$ " Brix
$\underset{H^{\circ}=}{\mathrm{L}^{\circ}=} \quad$ " Beaumé for liquids $\underset{\text { " }}{\text { lighter }}$ than water

+ Signs for liquids $\underset{\text { lighter }}{\text { lign }}$ theavier water.


## METRIC EQUIVALENTS

1 Meter $=3.281$ feet $=1.0936$ yards.
1 Centimeter $=0.3937$ inches $=0.01$ meter.
1 Kilometer $=1000$ meters $=0.6214$ miles.
1 Kilogram $=2.205$ pounds $=9.806 \times 10^{5}$ dynes.
1 Gram $=0.03527$ ounces $=15.43$ grains.
1 Hectare $=2.471$ acres.
1 Liter $=0.2642$ gallons.
1 Kilogram per sq. centimeter $=14.223 \mathrm{lbs}$. per sq. in.
1 Metric Ton $=1000$ kilograms $=2205 \mathrm{lbs}$.
1 Pound $=453.59243$ grams.

## MATHEMATICAL FORMULAE

$\pi=$ Circumference of a circle $\div$ diameter $=3.141592$.
ARC $1^{\circ}=0.0174533 \times$ radius $=0.0174533$ radian.
$1^{\prime}=0.0002909 \times$ radius $=0.0002909$
" $1^{\prime \prime}=0.000004848 \times$ radius $=0.000004848$ radian .
" $57^{\circ} .296^{\circ}=$ radius $=57^{\circ}-17^{\prime}-44^{\prime \prime} .806=1$ radian.
60 seconds (") $=1$ minute (').
60 minutes $\left({ }^{\prime}\right)=1$ degree $\left({ }^{\circ}\right)$.
90 degrees $\left({ }^{\circ}\right)=1$ right angle.
Area of a Triangle $=$ base $\times$ height $\div 2=\frac{\sqrt{s}(s-a)(s-b)(s-c)}{}$
where $a b c$ are the lengths of the sides and $s=\frac{a+b+c}{2}$
Area of a Parallelogram or of a rectangle $=$ base $\times$ height.
Area of a Circle $=\frac{\pi(\text { diameter })^{2}}{2}=\underset{\pi}{2}=0.7854 \times(\text { dadius })^{2}$ diameter $^{2}$
Volume of a Sphere $=\frac{\pi(\mathrm{D})^{3}}{6}=0.5236 \times \mathrm{D}^{3}(\mathrm{D}=$ Diameter $)$.
Area of an Ellipse $=\frac{\pi}{4} \times$ small diameter $\times$ large diameter.
Volume of a Prism $=$ area of base $\times$ height.
Volume of a Pyramid $=$ area of base $\times$ height $\div 3$.
Area of a Parabola $=\frac{2}{3}$ base $\times$ height.
Volume of Prismoid $=1 / 6$ th height $\times$ (Sum of top and bottom areas and 4 times area of midway section).

Volume of Ellipsoid $=\frac{4}{3} \pi \times a \times b \times c=4.188756 \times a \times b \times c$ where $a, b, c$ are the semiaxes.
Volume of Paraboloid (of revolution) $=\frac{1}{2}$ area of base $\times$ height.
Volume of Spherical Segment $=\frac{\pi}{6} \times(\text { height })^{3}+\frac{1}{2}$ area of base $\times$ height.

## SOME MONEY EQUIVALENTS IN GOLD

1 Pound Sterling (Columbia, Great Britain and Colonies) $=\$ 4.8665=$
1 Libra (Peru).
1 Franc (France, Belgium, Switzerland) $=\$ 0.1929=1$ Lira (Italy) $=$
1 Peseta (Spain) $=1$ Drachma (Greece) $=1$ Markka (Finland) $=$
1 Leu (Roumania) $=1$ Dinar (Servia) $=1$ Bolivar (Venezuela) $=$
\$0.1929.
1 Yen (Japan) = 1 Peso (Mexico) $=\$ 0.4984$.
1 German Mark $=\$ 0.2381$.
1 Balboa (Panama) $=1$ Cordoba (Nicaragua) $=\$ 1=1$ Dollar (U.S.A., Canada, British Honduras, Liberia, Santo Domingo).

| ROOT TABLE |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| N. | $\sqrt{\mathbf{N}}=\mathrm{N}^{1 / 2}$ | $\sqrt{\mathbf{N}} \times \mathrm{N}^{1 / 3}$ | $\mathrm{N}^{3 / 2}$ | $\mathrm{N}^{2 / 3}$ |
| 0.0001 | 0.01 | 0.04642 | 0.000001 | 0.002154 |
| 0.001 | 0.03162 | 0.1 | 0.00003162 |  |
| 0.01 | 0.1 | 0.2154 | 0.001 | 0.04642 |
| 0.1 | 0.3162 | 0.4642 | 0.031623 | 0.2154 |
| 1. |  |  |  |  |
| $\stackrel{10}{10}$ | ${ }_{10} 3.162$ | ${ }_{4}^{2.1542}$ | ${ }_{1000}^{31.6228}$ | ${ }_{21.54}^{4.646}$ |
| 100. 1000. | ${ }_{31.62}^{10 .}$ | ${ }^{4.642}$ | ${ }_{31622.8}^{1000}$ | ${ }_{100}^{21.54}$ |
| 10000. | 100. | ${ }_{21.54}$ | 1000000. | 464.16 |

Numbers between any two successive numbers of the first column of the table have corresponding roots between those given in the table.

Multiplying or dividing any number by 100 , multiplies or divides its square root by 10 , and its three-halves power by 1,000 . Multiplying or dividing any number by 1,000 , multiplies or divides its cube root by 10 and its two-thirds power by 100.

|  |  |  |  |  | $\infty$ | － |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 1010 ¢ | 0 | $\stackrel{\rightharpoonup}{*}$ | －0¢ | $\stackrel{\infty}{\sim}$ | －${ }_{8}^{\infty}$ | $\stackrel{\infty}{\sim}$ | Non |
| $\varepsilon$ | 1010 ゼ¢ | ${ }^{\circ}$ | 8 | ホN | E | － | ¢ั | ${ }_{\infty}^{\infty} 0$ |
| － | サ10 | e | － | － | 上 | O－ | ¢10 | $\infty$－ |
| $\stackrel{+}{0}$ | 10101010 | 0 | $\bigcirc$ | NNN | $\stackrel{\square}{0}$ | $\infty \times$ | $\infty$ | $\infty$ |
| ${ }_{0}$ | $000^{\circ} 0^{\circ}$ | ${ }^{\circ}$ | $\stackrel{\circ}{\circ}$ | $0^{\circ}{ }^{\circ}$ | $\bigcirc$ | 00 | －0 | $0^{\circ}$ |
| E | ｜｜｜｜｜｜｜｜｜｜ | 11 | 11 | 11 | 11 | 1111 | 1111 | 1 |
| O | －${ }^{-1}$ | $\stackrel{-1}{+}$ |  | －r |  |  |  |  |
| $\bigcirc$ |  |  | $\stackrel{\sim}{\square}$ | $\cdots$ | $\underset{\mathrm{N}}{\mathrm{o}}$ | ＋ |  |  |



FRACTION TABLE




䔍

［84］

ELEMENTARY SLIDE RULE MANUAL

## INDEX

Page
D-Scale . . . . 25, 34, 38, 40, 42, 44Duplex Slide Rule
64, 66 (F. 27), 67, ..... 79
Engineering Data ..... 82
Estimating ..... 51
$\mathrm{E}^{+\times}$Scale ..... 75, 76, 77
$\mathrm{E}^{-1}$ Scale ..... 75, 76, 77
Folded Scales ..... $73,74,75$
Formulae ..... 81-84
Fractional Power . . 60, 61, 78,
Fractions
Computation ..... 30, 36
Table ..... 85
Fuller Slide Rules ..... 67, 68
Geometric Scales ..... 67, 69
Graduated Length of a Rule ..... 73
Graduated Length of a Scale ..... 72, 73
Graduations ..... 9
Gravity-Acceleration ..... 47, 82
Gunter's Line ..... 63
Hairline ..... 5, 68, 69
Half-Length Scales ..... 73,74
Helical Scale....67, 68 (F. 31),75
History of Slide Rule ..... 63
Horse Power ..... 46, 82
Hyperbolic Functions ..... 75, 80
Inaccuracy Percentual.. 53, 62, ..... 80
Index Line ..... 73
Indicator ..... 63, 68, 69
Infinite Logarithmic Scale 72 (F. 40), 73,Interchangeability of Indices.17, 25
Interest Compound ..... 52
Simple ..... 52
Inverted Scale ..... 70, 71, 72, 73
I-Scale ..... 70, 71, 72, 73
Kilowatt ..... 46, 82
K-Scale ..... 73, 74 (F. 43)
Last Figure of a Multiplica-tion48, 54
Length of a Scale-Graduated ..... 73
Limitations
Mannheim ..... 53
Ritow ..... 62
Logarithmic Scale ..... 71
Logarithms-Scale of 70 (F. 35-L), ..... 71
Log-Log Scale. . . . 64, 75, 76, ..... 77
L-Scale 70 (F. 35-L), 71
Magnifier ..... 69

## INDEX—Continued

Page
Manifold Slide Rule 54, 62, 70 (F. 36), 71, 79
Mannheim Slide Rules. ..... 5,53
Markings-Special ..... 46
Mathematical Formulae ..... 83
Measures and Weights ..... 81
Merchants' Log-Log Slide Rule 54, 62, 75, 76 (F. 48, 49), 79, 80
Minutes ..... 46
Multiphase Duplex Slide Rule. ..... 79
Multiphase Slide Rule. .71, 72, 79
Multiplication ..... 31
By Series ..... 35
Mannheim Type. ..... 35
Ritow Type ..... 57
Payroll Computation ..... 48, 49
Payroll Change ..... 49
Per Cent Computation ..... 51, 52
Percentual Inaccuracy. 53, 62, ..... 80
Pi ( $\pi$ ) ..... 46, 83
Piecework Computation. ..... 36, 49
Point-Decimal ..... 19
Powers and Roots
$40,44,59,60,84$
Principles ..... 23
Problems-Special ..... 48
Proportions 23, 31, 33, ..... 57
Ratio and Proportion. .31, 33 ..... 57
Reading a Slide Rule ..... 9
Reciprocal Readings ..... 77
Reciprocals-Scale of
70, 71, 72 ..... 73
Reversed Scale. ..... 70, 71, 72, 73
Ritow Type ..... 55, 56
Cylindrical ..... 62, 67, 68, 80
Manifold
. .54, ..... 1, 79
Merchants' Log-Log . . . . 54, 62, 75, 76 (F. 48, 49), 79, 80
Simplex 64, 68 ..... F. 32), 69
Students' ..... 55, 56
Roots
Cube ..... 44, 60
Square ..... 40, 59
Table ..... 84
R-Scale ..... $55,56,74,75$
Runner ..... $63,68,69$Page
Scale
Of Logarithms, 70 (F. 35-L), 71
Of Sines ..... 70, 71
Of Tangents ..... 70, 71
Logarithmic ..... 7, 70, 71
Log-Log ..... 64, 75, 76, 77
Uniformly Graduated. ..... 70 (F. 35-L), 71
Seconds ..... 46
Selecting a Slide Rule ..... 79
Series
Division ..... 37, 57
Multiplication ..... 35, 57
Setting the Slide or Indicator to a Given Reading ..... 17
Simplex Slide Rule ..... 64, 68 (F. 32), 69
Sine Scale ..... 70, 71
Single Fold Scale
72 (F. 40), 74 (F. 44, 45), 75
Single Length Scale 73, 74 (F. 41)
Slide ..... 2, 5, 69
Special Markings. ..... 46
Special Rules ..... 80
Square Root ..... 40
Ritow ..... 59
Squares
Mannheim ..... 40
Ritow ..... 59
S-Scale ..... 70, 71
Statistics-Business ..... 49
Subdivision of Scales ..... 5, 9, 11
Tables ..... 81-85
Tangent Scale ..... 70, 71
Technical Rules. ..... 79
Ten-Fold Scale ..... 75, 76 (F.48)
Thacher Slide Rule, 66 (F. 29), 67Three-Fold Scale.
70 (F. 36), 75, 76 (F. 47)
Three-Halves Powers. ..... 45,84
Trigonometric Scales. ..... 70, 71
Triple-Length Scale
70 (F. 36), 75, 76 (F. 47)
T-Scale ..... 70,71
Two-Fold Scale . . . .55, 56, 74, ..... 75
Two-Thirds Powers. ..... 45
Weights and Measures ..... 81

## UNIVERSITY OF CALIFORNLA LIBRARY

This book is DUE on the last date stamped below.
Fine schedule: 25 cents on first day overdue 50 cents on fourth day overdue One dollar on seventh day overdue.


## YC 32572

 Mट58095

THE UNIVERSITY OF CALIFORNIA LIBRARY

